Collisional Relaxation of Fine Velocity Structures in Plasmas

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The existence of several characteristic times during the collisional relaxation of fine velocity structures is investigated by means of Eulerian numerical simulations of a spatially homogeneous force-free weakly collisional plasma. The effect of smoothing out velocity gradients on the evolution of global quantities, such as temperature and entropy, is discussed, suggesting that plasma collisionality can locally increase due to velocity space deformations of the particle velocity distribution function. These results support the idea that high-resolution measurements of the particle velocity distribution function are crucial for an accurate description of weakly collisional systems, such as the solar wind, in order to answer relevant scientific questions, related, for example, to particle heating and energization.

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The description of collisional effects in plasmas represents historically a huge scientific topic in which a significant numerical and theoretical effort has been made even in recent years [1–8]. In a weakly collisional plasma, such as the solar wind, collisions are usually considered far too weak to produce any significant effect on the plasma dynamics [9]. However, the estimation of collisionality is often based on the restrictive assumption that the shape of the particle velocity distribution function (VDF) is close to Maxwellian [10–12]. On the other hand, kinetic simulations [13–20], as well as in situ spacecraft measurements in the solar wind (SW) [12,21–24], indicate that marked non-Maxwellian features develop in the three-dimensional VDFs (temperature anisotropies, particle beams, ringlike modulations, etc.). Fine velocity structures naturally form in a kinetic plasma when an initial inhomogeneity is let free to evolve (ballistic effect) [25,26]. Nonlinear wave-particle interactions can lead in addition to larger scale velocity structures possibly taking part in a kinetic turbulent cascade and/or to instabilities. Since collisional effects increase with the velocity gradients of the VDF [27–33], the collisionless hypothesis may locally fail.

Velocity space structures can store the VDF free energy [34], which is then available for wave production through microinstabilities or for heating production due to collisional thermalization, which generates a degradation of information, i.e., an entropy production. Hence, investigating the role of collisions on small scale velocity space structures is relevant for understanding how collisionless wave-particle interactions compete with collisional processes and how efficiently collisions can be enhanced by the presence of fine velocity space gradients and play a significant role in converting ordered energy into heat. To highlight this effect, it is mandatory to adopt nonlinear collisional operators where the strength of the collisional terms depends on the VDFs shape. Up to now, collisional effects have been almost always modeled through simplified operators [35–40], which are linear or assume a reduced dimensionality in velocity space.

On the contrary, we modeled collisions through the full Landau operator, which is a nonlinear integro-differential operator of the Fokker-Planck type, has good conservation properties, and satisfies an H-theorem for the Gibbs-Boltzmann entropy [27,28]. The Landau operator introduces an upper cutoff of the integrals at the Debye length to mimic the effects of the time correlations due to the eigenmodes and so avoid divergence at large scale. A more general treatment of collisions (Balescu-Lenard operator [29,30]), which introduces the eigenmodes in a more consistent way through the dispersion equation, is much more difficult to manage numerically.

In this Letter, we discuss the collisional dissipation of non-Maxwellian features in the particle VDFs in a weakly collisional plasma, by means of Eulerian numerical simulations. Because of the nonlinear nature of the Landau operator, the analytical treatment as well as the self-consistent numerical simulations of the Landau operator in 6D phase space are difficult goals to achieve yet. Thus, we decided to address the collisional relaxation of a spatially homogeneous force-free plasma and to model collisions between particles of the same species through the following (dimensionless) collisional evolution equation for the particle distribution function $f(\mathbf{v})$:

$$\frac{\partial f(\mathbf{v})}{\partial t} = \pi \left( \frac{3}{2} \right) \int d^3v' U_{ij}(\mathbf{u}) \frac{\partial f(\mathbf{v})}{\partial v_i} \frac{\partial f(\mathbf{v})}{\partial v_j},$$

(1)

being $f$ normalized such that $\int d^3v f(\mathbf{v}) = n = 1$ and $U_{ij}(\mathbf{u})$

$$U_{ij}(\mathbf{u}) = \delta_{ij} u_i^2 - u_i u_j,$$

(2)
where $u = \mathbf{v} - \mathbf{v}'$, $u = |u|$ and the Einstein notation is introduced. In Eq. (1), and from now on, time is scaled to the inverse Spitzer-Harm frequency $\nu_{SH}^{-1}$ [10] and velocity to the particle thermal speed $v_{th}$. Details about the numerical solution of Eq. (1) can be found in Ref. [3].

In the first part of the present Letter, we consider the mutual effect of a local deformation of the particle VDF (a plateau) and the global temperature anisotropy, by comparing the evolution of two initial VDFs:

$$f_1(v) = C_1 f_{M,T_0}(v_x) f_{M,T_0}(v_y) f_{p,T_0}(v_z),$$

$$f_2(v) = C_2 f_{M,T_0}(v_x) f_{M,T_0}(v_y) f_{M,T_0}(v_z),$$

where $C_1$ and $C_2$ are normalization constants. The total temperature $T$, where $T = v_{th}^2$ in dimensionless units, is given by $T = (T_\parallel + 2 T_\perp)/3$ and $A = T_\perp/T_\parallel = 2$. Finally $f_{M,T_i}$ is a generic Maxwellian with temperature $T_i$ and $[2,4]$:

$$f_{p,T_0}(v_z) = f_{M,T_0}(v_z) - \frac{f_{M,T_0}(v_z) - f_{M,T_0}(V_0)}{1 + ([v_z - V_0]/\Delta V_p n_p)}$$

where $T_0 = 1$, $V_0 = 1.44$, $\Delta V_p = 0.5$, and $n_p = 8$. The function $f_{p,T_0}(v_z)$ is constructed in such a way to have a plateau of width $\Delta V_p$ around $v = V_0$, that is $f_{p,T_0}(v_z)$ is about null in the interval $V_0 - \Delta V_p/2 \leq v_z \leq V_0 + \Delta V_p/2$, being exactly zero at $v_z = V_0$.

It is worth to note that $f_2(v)$ is a bi-Maxwellian function, while $f_1(v)$ is Maxwellian in the perpendicular directions with a plateau centered in $v_z = V_0$ in the parallel direction. We also point out that $f_1(v)$ and $f_2(v)$ have the same temperature (second order moment) in each direction. Moreover, for the function $f_1(v)$, we reset the small mean velocity (=10^{-2}) produced by the presence of the plateau. The three-dimensional velocity domain is discretized with $N_{v_x} = N_{v_y} = 51$ and $N_{v_z} = 1601$ grid points. We point out that the resolution along $v_z$ has been increased significantly in order to resolve the short velocity scales associated with the presence of the plateau. Finally, the distribution function is set equal to zero for $|v_j| > v_{\text{max}} = 6 v_{th}$, being $j = x, y, z$.

As shown in Fig. 1(a), the time evolution of parallel and perpendicular temperatures of $f_1(v)$ (black solid line) and $f_2(v)$ (red dashed line) is clearly the same. On the other hand, the evolution of the entropy variation $\Delta S = S(t) - S(0)$ ($S = - \int f \ln f df$), reported in Fig. 1(b), displays significant differences. In particular, for $f_1(v)$ (black solid curve), the case in which a plateau is present, $\Delta S$ saturates at a larger level than that recovered for $f_2(v)$ (red dashed curve). In order to investigate the reasons of such different behavior of the entropy for $f_1(v)$ and $f_2(v)$, we performed a multieponential fit [41] of $\Delta S$ for the two cases, with the following curve:

$$\Delta S(t) = \sum_{i=1}^{K} \Delta S_i(1 - e^{-t/\tau_i}),$$

$\tau_i$ being the $i$th characteristic time and $K$ is evaluated through a recursive procedure.

From this analysis, we found that, while for the case of $f_2(v)$ [red dashed curve of Fig. 1(b)] $\Delta S$ shows an exponential growth with a single characteristic time ($\tau \approx 2 \nu_{SH}^{-1}$), for $f_1(v)$ [black solid curve of Fig. 1(b)], i.e., in the presence of a plateau, two different characteristic times are recovered: a fast characteristic time $\tau_1 = 0.14 \nu_{SH}^{-1}$ [indicated in Figs. 1(a) and 1(b) by a vertical blue dashed line] in which 25% of the total entropy growth is achieved, and a slow characteristic time $\tau_2 = 2.03 \nu_{SH}^{-1}$ during which the remaining 75% of the total entropy growth is observed. We argue that the existence of the characteristic time $\tau_1$ is due to the presence of the plateau, and in particular it is associated with the sharp velocity gradients in $f_1(v)$, while $\tau_2$ is related to the initial temperature anisotropy. In fact, as it can be seen in Fig. 1(c) where $f_1(v_x = v_y = 0, v_z)$ is plotted as a function of $v_z$ at $t = 0$ (black solid line) and at $t = \tau_1$ (red dashed line), the initial plateau is completely smoothed out by collisional effects in a time close to $\tau_1$, while from Fig. 1(a) one realizes that at $t = \tau_1$ the temperature anisotropy is still present.

To further support the idea that the presence of sharp velocity gradients in the particle VDF causes the entropy to grow over different time scales, we made an additional numerical experiment of collisional relaxation, considering a different initial condition for Eq. (1). This initial condition has been designed as follows. First, we performed a 1D-1V Vlasov-Poisson simulation (kinetic electrons and motionless protons) with high numerical
We emphasize that this VDF has the same temperature in each velocity direction but presents strong non-Maxwellian deformations along $v_z$, as shown in Fig. 2(a), which make the system far from equilibrium. The time history of $\Delta S$, obtained when using $f$ as initial condition for Eq. (1), is presented in Fig. 2(b). As in the previous simulations, the three-dimensional velocity domain in this case is discretized by $N_{v_x} = N_{v_y} = 51$ and $N_{v_z} = 1601$ grid points.

By analyzing the entropy growth through the same method of multieponential fit discussed previously, three characteristic times are recovered in this case, whose values are reported below, together with the corresponding percentage of entropy variation: (i) $\tau_1 = 3.5 \times 10^{-3} v_{SH}^{-1} \to \Delta S_1/\Delta S_{tot} = 13\%$, (ii) $\tau_2 = 1.3 \times 10^{-1} v_{SH}^{-1} \to \Delta S_2/\Delta S_{tot} = 42\%$, (iii) $\tau_3 = 4.9 \times 10^{-1} v_{SH}^{-1} \to \Delta S_3/\Delta S_{tot} = 40\%$.

Characteristic times $\tau_1$, $\tau_2$, and $\tau_3$ are indicated as red diamonds in Fig. 2(b). In Fig. 3, we plot $f$ as a function of $v_z$ for $v_x = v_y = 0$, at three different times $t = \tau_1$ (a), $t = \tau_1 + \tau_2$ (b), and $t = \tau_1 + \tau_2 + \tau_3$ (c): during the time $\tau_1$, steep spikes visible in Fig. 2(a) are almost completely smoothed out; at time $\tau_1 + \tau_2$ the remaining plateau region is significantly rounded off, only a gentle shoulder being left; finally, after a time $\tau_1 + \tau_2 + \tau_3$, the collisional return to equilibrium is completed for the most part (a small percentage $\simeq5\%$ of the total entropy growth is finally recovered for larger times and corresponds to the final approach to the equilibrium Maxwellian, indicated by red dashed lines in the three panels of Fig. 3).

Compared to the case shown in Fig. 1, here we recovered an additional extremely fast characteristic time ($\simeq10^{-3} v_{SH}^{-1}$), associated with the sharp velocity gradients of $f$ along $v_z$, while we did not detect the large characteristic time ($\simeq2 v_{SH}^{-1}$) associated with the temperature anisotropy in the previous case.

Numerical experiments discussed so far give a clear message: collisional dissipation of small velocity scales in the particle VDF occurs over different characteristic times, inversely proportional to the sharpness of the velocity gradients associated with those velocity scales. As we discussed above, these characteristic times can be significantly smaller than the Spitzer-Harm collisional time [10],...
this meaning that the presence of velocity gradients in fact speeds up the growth of the entropy of the system. This evidence suggests that when the particle VDFs exhibit small velocity scale deformations, the quasi-Maxwellian approximation, on which the Spitzer-Harm collisional evolution is based, is no longer appropriate.

In order to explore the implications of our results to the general case of the SW plasma, we performed our analysis on a three-dimensional proton VDF \( f_{sw}(v) \), obtained from the hybrid Vlasov-Maxwell [49] numerical simulations of SW decaying turbulence described in detail in Refs. [15–20]. As shown in Fig. 4(a), where the three-dimensional iso-surface plot of \( f_{sw} \) is reported, kinetic effects along the cascade make the VDF depart from the spherical shape of Maxwellian equilibrium and resemble a deformed potato. Then, having in mind to mimic low resolution VDF measurements by a real spacecraft, we fitted \( f_{sw}(v) \) with a tri-Maxwellian function \( \tilde{f}_{sw}(v) \) [Fig. 4(b)] and with a bi-Maxwellian function \( \hat{f}_{sw}(v) \) [Fig. 4(c)]. In order to point out the loss of physical information caused by not adequately resolving the sharp velocity gradients in the particle VDFs, the functions \( f_{sw}, \tilde{f}_{sw}, \) and \( \hat{f}_{sw} \) are used as initial conditions in three new simulations of Eq. (1), in which the velocity domain is now discretized by \( N_{v_x} = N_{v_y} = N_{v_z} = 51 \) grid points, as in the simulations in Refs. [15–20]. The results for the entropy growth of these new numerical experiments are reported in Fig. 5, where we show the time evolution of \( \Delta S \) for the VDFs \( f_{sw}(v) \) (black solid line), \( \tilde{f}_{sw}(v) \) (red dashed line), and \( \hat{f}_{sw}(v) \) (blue dashed line), respectively.

As for the previous cases discussed above, also here the time history of \( \Delta S \) is evidently affected by the presence of fine velocity scales and steep gradients in the particle VDF. Any fitting procedure, which smooths out the fine velocity structures, reduces the entropy growth: in fact, the simulation with the function \( \tilde{f}_{sw}(v) \) as initial condition displays a collisional entropy growth about 20 times smaller than that recovered for the case of the function \( f_{sw}(v) \). Moreover, through the multiexponential fit analysis performed on \( \Delta S \) for the simulation initialized with \( \hat{f}_{sw}(v) \), we found two characteristic times: a fast one \( \tau_0 = 0.20\nu_{SH}^{-1} \), in which 26% of the total entropy growth is achieved, and a slow one \( \tau_1 = 0.82\nu_{SH}^{-1} \), during which the remaining 74% of the total entropy growth is observed. By analyzing VDF iso-surface plots (not shown here) at different times in the simulation, we realized that after a time \( t = \tau_1 \) collisions have dissipated most of the sharp velocity gradients which were initially present in the VDF. We point out that, since the numerical resolution for this simulation is about 30 times smaller than in the previous case, sharp velocity gradients [as those shown in Fig. 2(a)] are not visible in the particle VDF, even though it displays significant non-Maxwellian features [see Fig. 4(a)]. Hence, the lack of velocity resolution presumably does not allow us to recover the extremely fast characteristic time \((=10^{-3}\nu_{SH}^{-1})\) in the evolution of \( \Delta S \), observed for the simulation initialized with the velocity profile in Fig. 2(a).

In this Letter, we discussed the role of the VDF fine velocity structures in enhancing the plasma collisionality. In particular, by means of Eulerian simulations of collisional relaxation of a spatially homogeneous force-free plasma, we have shown that the system entropy growth occurs over several time scales, which gets smaller as VDF gradients become steeper. We reported clear evidences that these gradients are dissipated by collisions in a time much shorter than that associated with global non-Maxwellian features, e.g., temperature anisotropies. This characteristic time may be comparable or even smaller than the instability growth rates invoked to explain the SW anisotropic VDFs [50,51] or than the nonlinear dynamics times, as recently discussed through a classical treatment of collisions [8].

We finally pointed out how the lack of resolution in the VDFs measurements mask a relevant part of physical

![FIG. 4. Iso-surface plot of the initial VDFs \( f_{sw}(v) \) (a), \( \tilde{f}_{sw}(v) \) (b), and \( \hat{f}_{sw}(v) \) (c), respectively.](image)

![FIG. 5. Entropy growth for the initial VDFs \( f_{sw}(v) \) (black line), \( \tilde{f}_{sw}(v) \) (red dashed line), and \( \hat{f}_{sw}(v) \) (blue dashed line), respectively.](image)
information hidden in the sharp velocity gradients of the non-Maxwellian VDFs, observed ubiquitous, for example, in the SW [21,24]. Future space missions, planned to increase both energy and angular resolutions of the VDFs measurements, will provide crucial insights for the long-standing problems of plasma heating and particle energization in the interplanetary medium.

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