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Excitation of nonlinear electrostatic waves with phase velocity close to the ion-thermal speed

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Received 30 June 2011, in final form 24 August 2011
Published 22 September 2011
Online at stacks.iop.org/PPCF/53/105017

Abstract
Recent hybrid Vlasov–Maxwell simulations predicted the presence of a significant level of electrostatic activity, consisting of ion-acoustic waves and a new branch of waves (called ion-bulk (IBk) waves), at short spatial scalelengths in the tail of the solar-wind turbulent cascade along the direction of the ambient magnetic field. These waves, driven by particle trapping processes, have phase speed comparable to the ion-thermal speed and acoustic dispersion. In this paper, the hybrid Vlasov–Maxwell code redesigned in the electrostatic configuration is used to reproduce the excitation of the IBk waves. An external driver electric field is applied to the plasma ions in order to create a population of trapped particles. This inhibits Landau damping that would otherwise suppress these low phase speed oscillations. The aim of this work is to show that the IBk waves, like ordinary linear modes, can be excited by general type drivers (even of low amplitude) applied to the plasma. Finally, Vlasov–Poisson simulations, with kinetic ions and Boltzmann electrons, are used to point out the limitations of the quasi-neutrality assumption, on which the hybrid model is based.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The study of the role of kinetic effects on the dynamics of a collision-free plasma is currently the subject of an extensive effort both for the case of interplanetary environments and of laboratory plasma systems. In the absence of collisional processes, kinetic effects, such as wave–particle resonant interactions, can provide efficient mechanisms that transfer energy from electric or magnetic fluctuations to the plasma and vice-versa. For example, in the case of the solar-wind plasma, where the mean-free path of the particles is of the order of 1AU, understanding of the kinetic dynamics would represent a very significant point in the study of the space plasma heating problem. This idea has motivated several experimental and theoretical works to consider the study of the collision-free plasma dynamics at the typical kinetic scales in the solar-wind of primary importance [1–3]. A key point of these analyses
is the identification of the fluctuations that carry the energy toward small wavelengths and eventually toward ‘dissipation’.

In this context, a considerable theoretical effort has been made by means of numerical simulations, focused on the analysis of the role of kinetic effects on the evolution of the system at short wavelengths. Recently, a hybrid Vlasov–Maxwell code [4] has been employed to study the dynamics of the solar-wind plasma at typical length scales of the order and below the ion inertial length \( \ell_i = \frac{c}{\omega_{pi}} \) (\( c \) begin the speed of light and \( \omega_{pi} \) the ion plasma frequency). Within this hybrid Vlasov–Maxwell model, the Vlasov equation is solved numerically for the ion species through a Eulerian algorithm, while the electrons are considered as a fluid. This numerical code has been used in a 1D-3V phase space configuration (one dimension in physical space and three dimensions in velocity space) in order to describe cross-scale effects across the ion inertial length in the evolution of the solar-wind turbulence in the direction parallel to a background magnetic field [5, 6]. The same approach has been used in 2D-3V configuration [7] in order to study the anisotropy of the turbulent spectra in the short-scale tail of the energy cascade.

These Eulerian simulations in 1D-3V and 2D-3V phase space configurations have shown that the longitudinal dynamics parallel to the magnetic field direction might act as a possible efficient channel for turbulence to develop toward small wavelengths and that the small-scale kinetic region of the longitudinal energy spectra is dominated by a significant level of electrostatic activity. The Fourier analysis of the numerical signals made it possible to identify two branches of electrostatic waves in the short-scale termination of the turbulent cascade. In addition to the well-known ion-acoustic (IA) waves, that could explain the space observations discussed in [8–10], there is also a new branch of nonlinear waves with dispersion relation of the acoustic type and phase velocity close to the ion-thermal speed, which have been previously dubbed ion-bulk (IBk) waves [5, 6].

As we have recently discussed in [11], the IBk waves are nonlinear electrostatic oscillations of the Bernstein–Greene–Kruskal type [12] that propagate with a phase velocity close to the ion-thermal speed. These oscillations are excited and sustained by the presence of a trapped particle population in the ion velocity distribution which inhibits Landau damping. These IBk waves are analogous to the so-called electron acoustic waves (EAWs), which are undamped nonlinear electrostatic fluctuations with phase speed close to the electron thermal speed. An extensive theoretical literature [13–16] focuses on the nature of the EAWs; moreover, their existence has been demonstrated through numerical simulations [17–19] and in laboratory experiments with nonneutral plasmas [20, 21]. We point out that the term ‘nonlinear’ used for these waves refers to the fact that they are generated and sustained by nonlinear trapping processes; nevertheless, these oscillations can exist even at low amplitude.

In this paper, we investigate the features of the IBk waves in detail and reproduce the excitation process numerically, following the same procedure adopted in [17–19] for the EAWs, i.e. using an external driver electric field that creates the population of trapped particles [22] in the vicinity of the wave phase speed. We discuss in detail the role of both ions and electrons in the propagation of these fluctuations thus giving an at least qualitative explanation for the numerically obtained dispersion relation. Moreover, from our numerical simulations, we note that, during the driving process, secondary instabilities of the beam-plasma type can develop that bring energy to high wavenumber electric field components. We then focus on the study of the long term nonlinear dynamics, paying particular attention to the generation mechanism of phase locked solitons due to the onset of the electrostatic secondary instabilities inside the IBk trapping vortices. The numerical experiments have been performed using the hybrid Vlasov–Maxwell code described in [4], redesigned in the electrostatic configuration. In the following, we will refer to this hybrid electrostatic model as the hybrid-Vlasov (HV) model.
Vlasov–Poisson (VP) simulations, where the kinetic dynamics of ions is retained while the electron response remains of the Boltzmann type, are used in the low phase velocity regime (compared with the electron thermal speed) in order to point out the limits of validity of the HV approach and, in particular, of the quasi-neutrality assumption on which the HV model is based.

This paper is organized as follows. In section 2, we briefly discuss the basic equations that describe the propagation of electrostatic waves within the HV model and the VP model and the solution of the wave dispersion relation in the presence of trapping regions of vanishing width in the ion velocity distribution. In section 3, we discuss the setup of the numerical simulations and in section 4 we present the numerical results of the HV simulations as well as the comparison with those of the VP simulations. We conclude in section 5.

2. Basic equations and wave dispersion relations

The propagation of electrostatic waves in unmagnetized plasmas may be treated in a one-dimensional phase space configuration. We point out that in the regime of low phase velocity fluctuations ($v_{ph} \approx v_{th} \ll v_{te}$) of interest here, the electron response in the VP model is of the Boltzmann type; under these conditions, the HV model differs from VP model only in the quasi-neutrality assumption versus a more realistic low-frequency electron response.

In the electrostatic approximation (in 1D-1V phase space configuration) and for massless electrons, the hybrid Vlasov–Maxwell set of equations presented in [4] reduces in dimensionless variables to

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + E \frac{\partial f}{\partial v} = 0; \quad E = -\frac{1}{n} \frac{\partial P_e}{\partial x}$$

(1)

Here $f(x, v, t)$ is the ion distribution function and $E(x, t)$ is the electric field; the ion density $n$ is obtained as the zeroth order velocity moment of $f$. The electron pressure $P_e$ is considered to be a function of the density $n = n_e = n_i$ (quasi-neutrality is assumed). In equations (1), times are scaled by the inverse ion plasma frequency $\omega_{pi}$, velocities by the ion-thermal speed $v_{ti}$, lengths by the ion Debye length $\lambda_{Di} = v_{ti}/\omega_{pi}$ and the electric field by $m_i v_{ti} \omega_{pi}/e$, where $e$ and $m_i$ are the ion charge and mass, respectively, and the initial values for the ion density $n_0$ and temperature $T_i$ are used to calculate the ion plasma frequency and thermal velocity. An isothermal equation of state for the electron pressure is assigned in order to close the above set of equations, which in scaled units is written $P_e = n T_e$.

Using the same dimensionless quantities, the VP equations read

$$\frac{\partial f_{\alpha}}{\partial t} + v \frac{\partial f_{\alpha}}{\partial x} + M_{\alpha} E \frac{\partial f_{\alpha}}{\partial v} = 0; \quad \frac{\partial E}{\partial x} = \sum_{\alpha} \sigma_{\alpha} \int f_{\alpha} \, dv$$

(2)

where $\alpha = i, e$, $M_i = 1$, $-m_i/m_e$, $\sigma_{\alpha} = +1, -1$, for ions and electrons, respectively.

As discussed in [6], by linearizing equations (1) and following the Landau prescription [23, 24], for weak wave damping (or amplification), the solution for the complex frequency of the fluctuations ($\omega \approx \omega_R + i \omega_I$) can be obtained by looking for the roots of the function

$$D(k, \omega) \simeq D_R(k, \omega_R) + i D_I(k, \omega_R) + i \omega \partial D_R(k, \omega_R)/\partial \omega_R,$$

where

$$D_R = -\frac{c_s^2}{n_0} \int \left( \frac{\partial f_0/\partial v}{v - v_{ph}} \right) \, dv + 1; \quad D_I = -\frac{\pi c_s^2}{n_0} \left. \frac{\partial f_0}{\partial v} \right|_{v_{ph}}$$

(3)

Here, $P$ indicates the Cauchy principal value, $n_0$ and $f_0$ are the equilibrium density and the equilibrium ion distribution function, respectively, $c_s = \omega_{pe} \lambda_{De}$ is the ion-sound speed [24], where $\lambda_{De}$ is the electron Debye length. In our dimensionless units both the ion-sound speed and the electron Debye length are equal to the square root of the electron to ion temperature ratio $\sqrt{T_e/T_i}$. 3
The same procedure for equation (2) gives \[24\]

\[
D_R = 1 - \frac{\omega^2_{ph}}{k^2 n_0^2} P \int \left( \frac{\partial f_0}{\partial v} \right) dv + \frac{1}{k^2 \lambda_{De}^2} \tag{4}
\]

\[
D_I = -\pi \sum a \frac{\omega_{pe}^2}{k^2 n_0^2} \left. \frac{\partial f_0}{\partial v} \right|_{v_\phi} \tag{5}
\]

where the term \(1/k^2 \lambda_{De}^2\) has been obtained making the assumption that the wave phase speed is much lower than the electron thermal velocity \(v_{te}\).

We point out that the expressions for \(D_R\) can be obtained by multiplying the right-hand side of equation (4) by a factor \(k^2 \lambda_{De}^2\) and then assuming \(k^2 \lambda_{De}^2 \ll 1\) (quasi-neutrality approximation); similarly, the expression for \(D_I\) is easily obtained by multiplying the right-hand side of equation (5) by a factor \(k^2 \lambda_{De}^2\) and then neglecting the contribution of the electrons (approximation of fluid electrons).

From equations (3) we see that, due to the quasi-neutrality assumption, the HV approximation can only describe the propagation of waves whose phase speed does not depend on the wavenumber or, equivalently, whose real part of the frequency depends linearly on the wavenumber (i.e. acoustic-like waves). From equations (3), for \(v_\phi \gg v_{ti}\) and for small \(T_i/T_e\), one can analytically solve for the zeros of \(D(k, \omega)\). This gives the real and the imaginary parts of the frequency for non-dispersive IA waves as a function of \(k\) (for details see \[24\]). We point out that the contribution of electrons to the wave damping cannot be recovered within this HV model. In this limit of large phase velocities (with respect to \(v_{ti}\)), the wave amplitude is weakly Landau damped as the damping rate is proportional to the velocity derivative of the distribution function calculated at the wave phase speed \[24\]; on the other hand, solving numerically the equation \(D(k, \omega) = 0\) in the regime of slow phase velocities shows that in the linear approximation oscillations with \(v_\phi \simeq v_{ti}\) would be suppressed after few wave cycles. Nevertheless, as discussed in [6, 17], by assuming that particle trapping processes can flatten the velocity distribution \[22, 24\] such that \(\left. \frac{\partial f_0}{\partial v} \right|_{v_\phi} = 0\), one recovers an extra undamped root with phase speed close to \(v_{ti}\).

For example, by solving numerically for the roots of equations (3) when \(\left. \frac{\partial f_0}{\partial v} \right|_{v_\phi} = 0\) (assuming a trapping region of vanishing velocity width) and with \(T_e/T_i = 10\) one finds two undamped solutions with dispersion relation of the acoustic form \(\omega_{IA} = 3.72k v_{ti}\) (IA waves) and \(\omega_{IBk} = 1.45k v_{ti}\) (IBk waves). More generally, the dependence of the wave phase speed on the temperature ratio \(T_e/T_i\) is displayed in figure 1; here, the red (gray) and the black curves represent the phase velocity of the undamped IBk and of the IA waves, respectively. We see that two branches are well separated at large \(T_e/T_i\), while the two curves coalesce at small \(T_e/T_i\). Finally, no solutions are found below a critical value of the electron to ion temperature ratio \(T^* \simeq 3.5\), meaning that our model does not admit this kind of solutions for values of the electron temperature close to the ion temperature.

The presence of a ‘nose’-looking feature at low values of \(T_e/T_i\) can be illustrated with a simple heuristic model. In fact, we can account at least qualitatively for the main features of IA and IBk waves by considering a single electron population and by imagining that the ion population is composed of two (sub)populations of ions with density \(n_1\) and \(n_2\) (\(n_1 + n_2 = n\)) corresponding to the ions that have velocities, respectively, smaller and larger than the wave phase velocity. Since the velocities of the ions in the first distribution are smaller than the wave phase velocity we can consider the first distribution as cold and conversely the second as hot. We take the response of electrons to both IA and IBk waves to be given by the standard Boltzmann response, in agreement with the second of equations (1). This electron response shields the oscillating electrostatic field created by the cold ion population \(n_1\), while the hot
Figure 1. Solution for the roots of equations (3) obtained by assuming zero damping. The black and red (gray) curves represent the phase speed of IA and IBk waves, respectively; the dotted–dashed curve indicates the analytical solution for IA waves $v_{\phi}/v_t = [c_2^2(1 + 3T_i/T_e)]^{1/2}$ (where a corrective term $3T_i/T_e$ has been retained), valid for large $T_e/T_i$.

ion population $n_2$ plays a shielding role similar to that of the electrons. A similar splitting was adopted, e.g. for electron waves in fusion devices in [25].

Note that also the mode frequency decreases as the number of ‘cold’ ions decreases. This keeps the ‘cold’ ion response important even when their number is reduced. For large values of $T_e/T_i$ the phase speed of the IA waves is much larger than the ion-thermal velocity and we can take $n_2 = 0$, thus recovering in the quasi-neutrality limit the standard dispersion relation of IA waves. In the case instead of IBk waves, the ion-thermal velocity is of the order of the phase speed of the waves and we can assume a splitting where $n_1$ and $n_2$ are of the same order. In this case the contribution of electrons to IBk waves can be neglected as long as the shielding contribution of the hot ions dominates over the shielding contributions of electrons, i.e. as long as $n_2/T_i > n_e/T_e$. As the ratio $T_e/T_i$ is reduced, the shielding contribution of electrons and of hot ions becomes comparable (taking $n_2 = n_1$ they would be equal for $T_e/T_i = 2$) and consequently the distinction between the IA and the IBk waves disappears and the two branches coalesce, as shown in figure 1. For smaller values of $T_e/T_i$ the combined shielding by the electrons and by the hot ions of the electric field generated by the perturbed cold ion population does not allow any oscillatory perturbation to propagate.

The same assumption of a trapping region of vanishing velocity width in the ion distribution function can be applied to the VP model in order to neglect the contribution of the ions to the imaginary part $D_i$ of the dielectric function in equation (5). For the sake of simplicity, one can also assume that the contribution of the electrons to the wave damping is negligible if $v_{\phi} \ll v_{te}$. Now we can solve for the zeros of equation (4) to find the dependence of the real part of the frequency on the wavenumber for undamped modes. The $\omega$–$k$ dispersion diagrams obtained for different values of $T_e/T_i$ are shown in figure 2. For very large $T_e/T_i$ ($T_e/T_i = 10^5$ in the top-left plot; this unrealistically large value is considered only for the sake of illustration), the contribution of the electrons in equation (4) at fixed $k$ disappears, i.e. $1/k^2\lambda_{De}^2 \rightarrow 0$. When $k\lambda_{De}$ is so large that the electron shielding is completely negligible, the electrons become the species that does not contribute while the ions are the mobile species, so that the shape of the dispersion relation is the same as that of the electrons with immobile ions (see, for example, [16]), but with interchanged labels. The $\omega$–$k$ curve displayed at the top-left plot in figure 2 is known as the ‘thumb’ diagram (see, for example, [17]); at high frequencies (fixed ions), the ‘thumb’ curve represents Langmuir waves and EAWs, while in the low-frequency

5
regime, displayed in figure 2, one obtains ion Langmuir waves (ILWs, black solid curve) and IBk waves (red (gray) solid curve). The HV model, whose solutions are indicated in figure 2 by the dotted–dashed lines, cannot describe the ILW, as they require relatively large charge separation effects. Moreover, as one should expect, the HV model agrees with the VP model for the IBk branch only in the small-wavenumber range, where the quasi-neutrality approximation is valid.

As the electron to ion temperature ratio decreases (see the top-right panel in figure 2), the ILW branch is replaced by the IA branch (black solid curve) and the ‘thumb’ becomes a ‘tear drop’ [26]. Again, the HV model agrees with the VP model only in the small-wavenumber range (for non-dispersive IA and IBk waves), but, while the HV solution extends to higher and higher wavenumbers, the VP solution displays a nose-like structure at a value of $k$ that depends on $T_e/T_i$. As shown in figure 2, when the value of $T_e/T_i$ is decreased the nose occurs at smaller wavenumbers and the tear drop becomes smaller and smaller, while the acoustic branches predicted by the HV model approach one another. Finally, when $T_e/T_i < T^{*}_{ei} \approx 3.5$ (not shown here) both the VP solution and the HV solution disappear.

We recall that the ‘tear-drop’ diagram is the typical $\omega$–$k$ diagram for a plasma column with finite radial geometry (for example a nonneutral plasma confined in a Penning trap by an axial uniform magnetic field [21]). In this case, the resulting ‘tear-drop’ diagram represents the so-called Trivelpiece–Gould modes [21, 27] and the EAWs. From this analogy, we see that a homogeneous infinite plasma of ions and Boltzmann electrons behaves as a single-species plasma column with a transverse radius equal to the Debye length.

### 3. Setup of the numerical simulations

In order to demonstrate the existence of the IBk waves we drive the plasma ions with an external electric field of relatively low amplitude, whose effect is to create a population of
trapped particles. Keeping the velocity width of the trapping region very small would allow us to compare the numerical results with the analytical predictions discussed in the previous section, obtained under the assumption of a trapping region of vanishing width. We point out that, under these conditions, Eulerian Vlasov codes are to be preferred with respect to the particle-in-cell codes, since the statistical noise intrinsic in particle-in-cell algorithms can mask the physical information when used to describe the evolution of low amplitude fluctuations.

The electrostatic HV simulations, used to make contact to previous works of solar-wind turbulence [5–7], follow the ion dynamics in the $x$-direction, for many plasma periods ($t_{\text{max}} = 4000$). The phase space domain for the simulation is $[0, L_x] \times [-v_{\text{max}}, v_{\text{max}}]$, where $v_{\text{max}} = 5$ and $L_x \simeq 126$, and it is discretized with $N_x = 256$ gridpoints in the spatial domain (we present also the results of simulations with $N_x = 1024$ for comparison) and $N_v = 1601$ gridpoints in the velocity domain. The initial ion distribution function is taken to be Maxwellian in velocity space and at $t = 0$ both ions and electrons have homogeneous densities $n_0 = 1$ (simulations where a small noise in the electron and ion densities has been imposed in the initial conditions did not give significantly different results). Periodic boundary conditions in physical space are imposed. The external driver electric field, used to trap resonant ions, is taken to be a standing wave $E_D(x, t) = E_D^{\text{max}} f(t) [\sin(kx - \omega_D t) + \sin(kx + \omega_D t)]$, where $f(t) = [1 + (t - \tau)/\Delta \tau]^n$, $E_D^{\text{max}} = 0.001$, $\tau = 1200$, $\Delta \tau = 600$, $n = 10$ and $k = mk_0 = m(2\pi/L) \simeq m(0.05)$. The use of a standing wave as an external forcing electric field is convenient in the case of a periodic system, as it allows naturally for a null total current with a periodic distribution function. The external driver electric field is applied directly to the ions in the Vlasov equation; this corresponds to simulate the interaction of the waves with the ions such as, for example, the ion-cyclotron interaction considered in the simulations of solar-wind turbulence in [5, 6], where the IBk waves were first observed.

The plasma response is studied as a function of the driver phase velocity $v_{\phi_D} = \omega_D/k$. The driver is turned on and off adiabatically; after the driving process, the amplitude $E_D$ of the external electric field is nearly zero again at $t_{\text{off}} \simeq 2000$. The value of the maximum driver amplitude $E_D^{\text{max}} = 0.001$ has been chosen taking into account that the characteristic time $\tau$ (trapping time) needed to create the trapped population in the velocity distribution is proportional, for a fixed $k$, to the inverse square root of the trapping electric field amplitude [22]; this means that decreasing the value of the driver amplitude increases the time needed for the formation of the trapping region and consequently increases the required simulation time. For the simulations described in this paper the driver is switched on for about one and a half particle trapping period.

Finally a VP numerical code, that solves equations (2) numerically, will be used in order to point out the limits of validity of the quasi-neutrality approximation assumed in the hybrid model. The VP simulations have been initialized with the same parameters used for the HV simulations. Moreover, in the VP simulations we used the value $m_i/m_e = 100$ for the ion to electron mass ratio, so that the scaled electron plasma frequency is $\omega_{pe} = 10$.

4. Numerical results

4.1. HV simulations

In this section, we consider HV simulations in which an external electric field with $k = k_0 = 0.05$ (we drive the spectral component $m = 1$, the one with the longest wavelength that fits in the simulation domain) and $v_{\phi_D} = 1.85$ is applied to the plasma ions, for $T_e/T_i = 10$. According to the theoretical predictions shown in figure 2, for these parameters the results of the HV simulations should be in agreement with those of the VP simulations, since charge separation
effects should be negligible for such a low value of the wavenumber. We also anticipate that $v_{\phi D} = 1.85$ is the driver phase velocity for which we obtained the largest plasma response, as will be discussed in more detail in the following.

In figure 3, we show the time evolution of the electric field calculated at a fixed point in the spatial domain. From this plot we see that during the driving process the amplitude of the electric oscillations suddenly increases at about $t = 1500$ and large amplitude impulsive spikes appear; the electric oscillations survive even after the external driver has been switched off ($t = t_{\text{off}} \simeq 2000$). Moreover, we note that the electric field signal in figure 3 is the result of the superposition of many frequencies. In fact, by separating the contribution of the different spectral components of the electric field, we observe the growth of many modes with larger and larger wavenumbers during the driving process of the $m = 1$ mode. This is seen in figure 4 (top plot), where we show (in semi-logarithmic scale) the time evolution of the spectral components $m = 1, m = 4$ and $m = 6$; we observe that modes with $m > 1$ start growing during the driving process and quite rapidly reach an energy level comparable to that of the mode $m = 1$. From the $\omega-k$ Fourier spectrum of the electric energy (bottom plot) we see that the spectral components of the electric field propagate with the same phase velocity $v_{\phi} \simeq 1.85$ and form an acoustic branch.

The fact that the modes with $m > 1$ grow exponentially in time suggests that the system has become unstable during the driving process. In order to provide a physical explanation to this phenomenon, we look at the ion distribution function at the time instants $t = 1200, 1220$, when the external electric field is still pumping energy into the system, the spectral components with $m > 1$ have started their exponential growth, but the $m = 1$ mode is still dominant (see vertical (red) dashed line in figure 4). In figure 5 (top row), we show the level lines of the ion distribution function in phase space at $t = 1200$ (a) and $t = 1220$ (b). These plots display the typical region of trapped particles moving at mean velocity equal to $v_{\phi} = 1.85$ in the positive $x$ direction (a similar structure is generated in the negative velocity range of the phase space, since we drive the system with a standing wave external electric field). In the bottom row of figure 5, we plot the velocity derivative $[(\partial f/\partial v)|_{v_{\phi}}$ of the ion distribution function calculated at $v = v_{\phi} = 1.85$ as a function of $x$ at the time instants corresponding to the phase space plots in the top row of figure 5. We notice that the external electric field has created regions of positive velocity slope; in particular, $(\partial f/\partial v)|_{v_{\phi}}$ has a positive and almost constant value inside the trapping region and remains locked in the vortical structure as time goes on. In order to show that the positive velocity slope region remains stable in time, in figure 6 we give the
At the top: time evolution of the spectral components $m = 1$, $m = 4$ and $m = 6$ of the electric field for $v_{\phi D} = 1.85$ and $T_e/T_i = 10$; at the bottom: $\omega$–$k$ spectrum of the electric energy. (1) superposition at different times of the ion distribution function calculated as a function of $v$ at the spatial point $x_m$ where at each chosen time $\partial f/\partial v$ is maximum, or, equivalently, at the center of the trapping region which propagates with velocity $v_{\phi}$. The times are taken inside the interval $1200 < t < 1400$, i.e. during the initial phase of the exponential growth of the high wavenumber components of the electric field. Figure 6 indicates that a well defined and stable bump is present in the vicinity of $v_{\phi}$ (the trapping region is indicated by vertical red (gray) dashed lines).

As a consequence of the generation of this bump the ion velocity distribution becomes unstable in the spatial regions where the ions are trapped in the wave potential well. In figure 7, we show four snapshots of the system evolution at four successive times $t = 1380$ (a), $t = 1400$ (b), $t = 1420$ (c) and $t = 1460$ (d). Each column in figure 7 displays the phase space trapping region in the positive velocity range (top plot) and in the negative velocity range (bottom plot) and the corresponding electric field (middle plot). It is clear from figure 7 that two well defined wave packets are generated at the spatial positions corresponding to the trapped particle regions. Each of these wave packets remains locked inside the vortical structure that has generated it. Thus the two wave packets propagate independently in opposite directions, their amplitude growing in time. Moreover, it is seen from figure 7(d) that the growth of the high wavenumber components leads to the formation inside the large-scale mother vortex of secondary vortical structures in phase space with short spatial scalelengths. From figure 7(d) one can easily estimate that the characteristic wavelength of these high wavenumber structures is $\simeq 0.2$, shorter than the Debye length $\lambda_{De} = \sqrt{T_e/T_i} \simeq 3.2$. At such scalelengths the quasi-neutrality assumption of the hybrid model breaks. In the next section,
we will analyze in detail the effects of charge separation on the generation of these structures, using VP simulations.

Based on these numerical results, one may try to explain the growth of the high wavenumbers components as an instability process triggered by the positive velocity slope of the ion velocity distribution at \( v \approx v_\phi \). In order to prove that the exponential growth of the
Figure 7. HV simulation. Level lines of the phase space ion distribution function in the positive/negative velocity range (top/bottom row) at $t = 1380$ (a), $t = 1400$ (b), $t = 1420$ (c) and $t = 1460$ (d); the corresponding electric field signal is shown in the middle row.

Spectral components with high wavenumbers is the consequence of the generation of unstable regions in phase space and that it is not due to the external field continuously pumping energy into the system, we repeated the simulation smoothly turning off the driver as soon as the positive slope in the ion velocity distribution is created (at about $t \simeq 900$), but before the growth of modes with $m > 1$. The results of this additional simulation (not shown here) confirm that the appearance of the high wavenumber components in the electric field signal is due to the generation during the driving process of unstable regions with positive velocity slope of the ion distribution function.

Equations (3) describe the dielectric response of a spatially homogeneous plasma. In the case of our simulations, the plasma is not homogeneous in space, but, as we discussed above, the unstable spectral components grow and generate local wave packets localized inside well defined spatial regions (the central portion of the trapped particle regions), where the velocity derivative of the ion distribution function calculated at $v_\phi$ has a positive and nearly constant value (see figure 5). Thus we may consider a ‘local approximation’ and evaluate the growth rate of the high wavenumber components of the electric field as if due to a secondary instability in a locally uniform plasma. The imaginary part of the wave frequency can be evaluated as [24] $\omega_I = -\frac{D_I(k, \omega_R)}{\partial D_R(k, \omega_R)/\partial \omega_R}$.

In order to compare the above expression of $\omega_I$ with the numerical results, we evaluate $D_I$ and $D_R$, using in equations (3) the proton velocity distribution obtained from the simulations as the time average $f_T$ (over the interval $T = [1200, 1400]$) of the velocity distributions displayed in figure 6. In particular, the Cauchy principal value of the velocity integral in the first of equations (3) has been evaluated numerically on a discretized uniform velocity grid through a standard ‘trapezoidal’ algorithm, using the velocity distribution $f_T$ defined above. This comparison is shown in figure 8: the stars represent the numerical growth rates of the first 40 electric field modes, while the dashed line is the theoretical prediction above. The evolution of the electric field components $m > 40$ is affected by the fake dissipation due to the numerical discretization. For values of the wavenumber $k \lesssim 1$ the numerical growth rate displays a linear dependence on $k$, in agreement with the theoretical expectation; for $k > 1$ this
linear dependence saturates and for high wavenumber electric components with $1 < k < 2$ the numerical results for the growth rate depart from the theoretical line.

Finally, we performed several simulations using different driver phase velocities in the range $0.4 < v_{\phi_D} < 4.5$, keeping fixed all other parameters, in order to characterize the plasma response to the electric field of the external driver. For each simulation we consider the electric energy $E = \int E^2(x, t)/2 \, dx$ and take the maximum value of $\dot{E}$ in the time interval $[2500, 4000]$ (after the driver has been switched off, until the end of the simulation). The results for the maximum of $\dot{E}$ as a function of the driver phase velocity are reported in figure 9 in semi-logarithmic scale. The solid line connecting the simulation results (stars) refers to a set of simulations with $T_e/T_i = 10$. Two peaks are clearly distinguishable in this plot: the first peak is located at $v_{\phi_D} \simeq 1.85$ and the second at $v_{\phi_D} \simeq 3.7$. These two values are consistent with the phase velocities of the IBk and of the IA waves as predicted analytically in the previous section for $T_e/T_i = 10$, even though the value $v_{\phi_D} \simeq 1.85$ is somewhat larger than the analytical estimate $v_{\phi(\text{IBk})} = 1.45$ for the IBk waves. The reason for this discrepancy is due first of all to the fact that in the simulations the driving electric field has produced a bump in the ion velocity distribution instead of a plateau, as discussed above. In addition, the trapping region generated
during the driving process has a small but finite velocity width (for example, in figure 7 one can see that the velocity width of the trapping region is $\simeq 0.4v_{ti}$). This effect of phase velocity shift due to the finite velocity size of the trapping region has already been discussed in [17] for the case of the EAWs.

The effect of the non-vanishing trapping region can also be seen in the resonance curve in figure 9 from the fact that the excitation of IBk and IA waves can be obtained in a wide band of driver phase velocities around the two maxima; the two peaks in the resonance curve would become thinner and thinner as the width of the trapping region in the velocity distribution becomes smaller and smaller and would reduce to a delta function for vanishing velocity widths, thus reproducing the analytical results of figure 1.

The dashed line in figure 9 represents the resonance curve in the case of a different set of simulations performed with a low value of the electron to ion temperature ratio $T_e/T_i = 1$. The analytical curve in figure 1 predicts no wave excitation at such a low value of $T_e/T_i$. Nevertheless, the numerical results show a significant (but lower than that observed for $T_e/T_i = 10$) plasma response around $v_{\phi D} \simeq 1.8$ even at $T_e/T_i = 1$. Again, this is a consequence of the non-vanishing trapping region in the velocity distribution that makes wave excitation possible in a wide band of driver phase velocities. This result shows that the trapped particle population allows for the existence of a low level of electric fluctuations at slow phase speed even when the electron temperature is comparable to the ion temperature.

4.2. Comparison between HV and VP simulations

In this section, we compare the results of the HV simulations with those obtained through VP simulations. In figure 10 (the same as figure 3 for the HV simulations) we display the time evolution of the electric field calculated at a fixed spatial point for $T_e/T_i = 10$, in the case of an external driver electric field with phase velocity $v_{\phi D} = 1.85$, from the VP simulation. From this plot, we notice that the electric field oscillations are composed of many frequencies, produced by the same instability process observed in the HV simulations. Nevertheless, the level of these VP fluctuations is somewhat lower than that shown in figure 3 for the HV simulations. To quantify this statement, we consider the ratio between the time averaged electric energy $\mathcal{E}_{\text{VP}}$ in the interval [2500, 4000], corresponding to the signal in figure 10 obtained from the VP simulation, and the same quantity $\mathcal{E}_{\text{HV}}$ for the signal in figure 3 from the HV simulation. We obtain $\mathcal{E}_{\text{VP}}/\mathcal{E}_{\text{HV}} \simeq 0.65$. 

![Figure 10](image-url)
Repeating the same VP simulation with the real value of \( m_i/m_e = 1836 \), we found an increase in the electric energy ratio \( \epsilon_{VP}/\epsilon_{HV} \simeq 0.77 \). Other VP simulations performed with different values of \( m_i/m_e \) show that the level of the electric response is correlated with the mass ratio. This behavior can be explained by noting that the effect of wave damping by the electrons, ruled out from the HV simulations, is possibly at work in the VP simulations and that it decreases as \( \sqrt{(m_e/m_i)} \) for IA type waves [24]. To clarify this point better, we repeated the same simulation using a Vlasov–Yukawa (VY) code [11], in which the Vlasov equation is solved numerically for the ions, while the electron response is taken to be the standard Boltzmann response (massless fluid electrons). The same description of the electron dynamics was already adopted in [29], where resonant excitation of nonlinear ion waves was investigated in fluid approximation. We point out that for the parameters chosen in our numerical experiments, the ratio between the electrostatic potential energy and the electron thermal energy is very small: \( e\phi_{\text{max}}/kT_e \approx 1.3 \times 10^{-2} \) for simulations with \( T_e/T_i = 1 \) and \( e\phi_{\text{max}}/kT_e \approx 2.5 \times 10^{-3} \) for simulations with \( T_e/T_i = 10 \) (\( \phi_{\text{max}} \) represents the maximum value of the electrostatic potential obtained in each simulation). Therefore, in the VY simulations the electron response is linearized as \( n_e \approx n_0(1 + e\phi/kT_e) \). The electron response remains of the Boltzmann type and linear also in the VP simulations for values of \( m_e/m_i \geq 100 \).

The ratio between the electric energy obtained from the VY simulation \( \epsilon_{VY} \) and that obtained from the HV simulation is \( \epsilon_{VY}/\epsilon_{HV} \approx 0.94 \). From the comparison between HV, VY and VP models, we conclude that the maximum electric response is obtained in the HV simulation and in the VY simulation, where \( m_i/m_e = \infty \), but the most realistic situation is obtained in the VP simulation with \( m_i/m_e = 1836 \). Due to the fact that a slight difference in the electric response has been recovered also between the results of HV and VY simulations, we investigated in detail the role of charge separation effects, ruled out from the HV model, but retained in the VY simulations (and also in the VP simulations).

The effects of charge separation come into play at length scales of the order of the Debye length \( \lambda_{De} \). Since the value of the Debye length in scaled units is given by \( (T_e/T_i)^{1/2} \), charge separation effects are independent on the ion to electron mass ratio. In order to describe their role in the system dynamics, in the following we will compare the results of HV simulations with those of VP simulations with \( m_i/m_e = 100 \).

During the instability process discussed in the previous section, quite rapidly high wavenumber modes are excited, for which the condition of quasi-neutrality, imposed in the HV simulations, is no longer satisfied. It can be expected that the dispersion produced by charge separation inhibits the development of the instability for large wavenumber modes. We recall that, as shown in figure 2, at \( T_e/T_i = 10 \) the theoretical dispersion relation obtained from the VP model predicts no wave excitation for \( k > 0.425 \), while the HV model does not predict any nose-type feature for high wavenumbers.

In order to confirm these considerations, in figure 11 we report the spectral energy of the electric field as a function of the wavenumber at \( t = 4000 \) for both the HV simulations (top plot) and for the VP simulations (bottom plot) performed with two different spatial resolutions \( N_x = 256 \) (red (gray) solid line in each plot) and \( N_x = 1024 \) (black line in each plot). From the top plot, we notice that for \( N_x = 256 \) the electric energy is limited by the spatial resolution and is dissipated around \( k \approx 1 \), while when the number of the spatial gridpoints is increased to \( N_x = 1024 \) the numerical dissipation occurs at \( k \approx 4.5 \). This shows that in the HV case the energy spectrum would extend to higher and higher wavenumbers (as predicted by the analytical calculations in figure 2), the only limit being the numerical spatial resolution. On the other hand, for the VP simulations the electric energy is limited around \( k \approx 0.3 \) for both the case with \( N_x = 256 \) and the case with \( N_x = 1024 \), far from the limit of the numerical dissipation. We point out that the value \( k \approx 0.3 \) is very close to the value of the wavenumber...
corresponding to the Debye length, $k_{De} \simeq 1/\sqrt{T_e/T_i} \simeq 0.316$ (indicated in figure 11 by the vertical dashed line), showing how the effects of charge separation inhibit the transfer of energy toward high wavenumbers.

4.3. Vortex-merging process and late time evolution

To conclude our analysis, following the investigation of vortex merging in EAWs [17, 28], we looked for a similar phenomenon in the case of IBk waves, i.e. we studied the stability of the IBk waves against decay toward long wavelengths. In the case of the EAWs, it has been shown [17, 28] that the excited EAW oscillates at a nearly constant amplitude long after the driver is turned off, provided the EAW has the largest wavelength that fits inside the simulation domain. Otherwise, the excited EAW decays to a longer wavelength EAW.

In phase space, this decay to longer wavelength appears as a tendency of the vortex-like trapped particle populations to merge. In order to investigate the possibility of a decay instability also for the IBk waves, we repeated the same simulations both with the HV code and with the VP code, driving the $m = 2$ mode instead of the $m = 1$ mode. The system evolution is followed up to a time $t = 5500$ and its dynamics is found to be qualitatively similar to that discussed in the previous sections for the case $m = 1$. During the driving process of the mode $m = 2$, high wavenumber modes ($m > 2$) are excited, generating secondary vortical
structures in phase space inside the main large-scale structures. These secondary vortices tend to merge in time, but no vortex merging and energy transfer is observed from the mode $m = 2$ to the mode $m = 1$ (i.e. inverse cascade) in both HV and VP simulations. We emphasize that the same dynamics of the secondary vortical structures is recovered in long time ($t > 4000$) simulations where the mode $m = 1$ is driven by the external forcing.

In figure 12, we show how the phase space looks like at $t = 5500$ in the HV case (left column) and in the VP case (right column). Since we drove the mode $m = 2$ with a standing wave, four trapped particle structures are observed in phase space, two of them (top plot) propagating in the positive $x$ direction and the other two (bottom plot) in the negative $x$ direction. In the middle-left plot of figure 12, where we show the electric field as a function of $x$ at $t = 5500$, we observe the formation of spatial soliton-like signals locked in the trapping regions. We emphasize that these are extremely long-lived structures (at $t = 7000$ they are still stable). The same evolution is recovered for the VP simulations (middle-right plot), the only difference being that in the HV case the characteristic length of the soliton-like structures is limited only by the numerical spatial resolution (in principle, the larger $N_s$ is used, the shorter length-scale structures are produced), while for the VP simulation the characteristic thickness of the soliton-like centers is limited to something of the order of $\lambda_{Dx}$ by dispersive effects ($1 + k^2\lambda_{Dx}^2$) related to the Debye shielding, as discussed above.
5. Summary and conclusions

In this paper, we have discussed the excitation by an externally applied electric field of a new branch of nonlinear electrostatic waves, the ion-bulk waves, driven by the particle trapping process and propagating at phase speed close to \( v_{ti} \). We have analyzed the results of hybrid-Vlasov numerical simulations and we have used Vlasov–Poisson (with different values of the ion to electron mass ratio) and Vlasov–Yukawa simulations to point out the limitations of the quasi-neutrality assumption on which the hybrid model is based. The ion-bulk waves were first detected in hybrid Vlasov–Maxwell simulations of short-wavelength turbulence in the solar wind [5–7].

Here, we showed that the IBk waves can be excited by applying an external electric field that creates a population of trapped particles, thus preventing these low phase velocity waves to be Landau damped. The excitation process of the IBk waves is analogous to that of the so-called electron acoustic waves. Once the trapped particle population has been created during the driving process, successful excitation of IBk waves is obtained provided the driver is applied with a phase velocity consistent with the theoretical predictions discussed in section 2 of this paper. We note that during the driving process, even for small amplitude of the external driver, a small bump, instead of a flat plateau, is generated in the ion velocity distribution in the vicinity of the wave phase speed. This produces unstable phase space regions and triggers the exponential growth in time of larger and larger wavenumbers modes.

From these results, we conclude that in the hybrid Vlasov–Maxwell simulations of solar-wind turbulence presented in [5–7] the ion-cyclotron waves play the role of an external driver that creates a diffusive velocity plateau in the ion velocity distribution through resonant interaction with ions [30]. Once the velocity plateau has been created, the longitudinal IBk channel is open and becomes available for turbulence to carry the energy from large toward small wavelengths.

Using Vlasov–Poisson and Vlasov–Yukawa simulations we realized that the level of the electric response to the external driver electric field shows correlation with the value of the ion to electron mass ratio. The hybrid-Vlasov and the Vlasov–Yukawa simulations, for which \( m_i/m_e \to \infty \), give a slightly enhanced response with respect to the real case \( m_i/m_e = 1836 \). Moreover, in the Vlasov–Poisson and Vlasov–Yukawa simulations the instability triggered by the formation of a positive slope region in the velocity distribution of ions during the driving process is inhibited by charge separation effects. These effects prevent the transfer of energy toward wavenumbers larger than \( k_{De} = 1/\lambda_{De} \). As a consequence, the characteristic spatial length of the soliton-like structures shown at late times by the Vlasov–Poisson simulations is larger than the spatial length of the corresponding structures observed in the hybrid-Vlasov simulations. This indicates that the quasi-neutrality assumption, on which the hybrid model is based, is violated during the system evolution.

Acknowledgments

We thank T O’Neil, F Driscoll, A Kabantsev, F Anderegg and B Afeyan for fruitful discussions. We thank the INAF Key-Project 2010 and the Vlasov project supported by the EU DEISA Extreme Computing Initiative (EU FP6 project RI-031513 and FP7 project RI-222919). F V was partially supported by CNISM, UdR of Pisa, Italy.

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