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Cite as: Phys. Plasmas 26, 072301 (2019); https://doi.org/10.1063/1.5100125
Submitted: 15 April 2019 . Accepted: 06 June 2019 . Published Online: 12 July 2019

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Published Online: 12 July 2019

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ABSTRACT
Kinetic simulations based on the Eulerian Hybrid Vlasov-Maxwell (HVM) formalism permit the examination of plasma turbulence with a useful resolution of the proton velocity distribution function. The HVM model is employed here to study the balance of energy, focusing on channels of conversion that lead to proton kinetic effects, including growth of internal energy and temperature anisotropies. We show that this Eulerian simulation approach, which is almost noise-free, is able to provide an accurate energy balance for protons. The results demonstrate explicitly that the recovered temperature growth is directly related to the role of the pressure-strain interaction. Furthermore, analysis of local spatial correlations indicates that the pressure-strain interaction is qualitatively associated with strong-current, high-vorticity structures although other local terms—such as the heat flux—weaken the correlation. These numerical capabilities based on the Eulerian approach will enable a deeper study of transfer and conversion channels in weakly collisional Vlasov plasmas.

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I. INTRODUCTION
The space physics community has become increasingly aware that plasma turbulence has a significant impact on the observed properties of solar wind and other accessible space plasmas.1–5 Given that the cascade rate toward smaller scales is measured 3–5 and that an addition of proton internal energy is required in order to explain the observed temperature evolution within the inner heliosphere,6,7 it is natural to study the “plasma dissipation” that must be present to reconcile these observations. Accordingly, kinetic dissipation of turbulence in weakly collisional plasmas has become a very active topic throughout heliospheric physics and several standard approaches have been employed. One approach is to assume that the notion of Ohmic dissipation carries over from the collisional case and therefore to adopt as a proxy for dissipation the electromagnetic work on particles, namely, \( j \cdot E \) (with \( j \) being the electric current density and \( E \) the electric field).\(^6\) Another definition, especially adopted in simulation works, regards the role of interparticle collisions, which introduce irreversibility into the system.\(^6,10,11\) Within this viewpoint, dissipation is associated with an entropy growth. Still different approaches study the dissipation related to peculiar mechanisms. Some examples are the dissipation of particular wave modes, such as kinetic Alfvén12,13 and whistler waves,17,18 and the heating related directly or indirectly to magnetic reconnection.19–24 Magnetic reconnection, in particular, plays a decisive role in energizing particles and dissipating energy in turbulent plasmas.25

In each of these approaches, there is clearly an opportunity for physical insights, even if each approach may not capture all mechanisms that operate or their mutual interactions. The concept of dissipation here advanced—and implemented in the analysis of hybrid Vlasov-Maxwell (HVM) simulation of plasma turbulence—differs in that it is based on the premise that pathways to dissipation may be identified based on separations of the effects of transport, scale transfer, and energy conversion. In particular, one may uniquely characterize the conversion into internal energy although this process is not related to an entropy growth.

Beginning with the HVM system, it is straightforward to derive energy balance relations,\(^26\) which may be further analyzed to identify...
the basic channels of energy exchange \(^{27,28}\) and the evolution of the pressure tensor. \(^{29,30}\) One basic finding, reviewed in more detail below, is that the energy exchanged between the electromagnetic field and the proton bulk motions is due to the interaction of the electric field with the proton charge flux (current). Meanwhile, the flow kinetic energy may be exchanged with the internal energy through the combined action of the pressure-dilatation and the pressure-strain interactions.

In the Vlasov energy balance, one also finds several energy transport terms that—although being quite important in the dynamics—do not contribute, on average, to converting energy from one form to another. In particular, transport terms cannot produce internal energy. These developments are based on elementary properties of the Vlasov equation, but their significance for collisionless turbulence has not always been fully recognized. Based on this taxonomy, one may proceed to analyze the contributions to these channels of energy conversion and transfer employing a number of approaches: by global averaging, by examining distribution functions, by computing joint distributions and correlations, and by studying real space maps to identify coherent structures. \(^{27,28}\) Some of these analyses have also been implemented in Magnetospheric MultiScale (MMS) mission observations in the terrestrial magnetosheath. \(^{31}\) This program of analysis of energy conversion/correlations, and by studying real space maps to identify coherent structures will be undertaken for HVM in a subsequent study.

This paper is organized as follows: Sec. II introduces the theoretical model and the numerical setup of the numerical simulation performed here by using HVM turbulence simulation. It is notable that, for HVM simulation, it is possible to reconcile a total energy balance with good accuracy, an undertaking not easily achieved using methods such as kinetic Particle-In-Cell (PIC) codes. One may also implement scale averaging \(^{27}\) to examine the locality of energy transfer and the relative contribution of conversion processes at different scales. \(^{33,34}\) This will be undertaken for HVM in a subsequent study.

This program of analysis of energy conversion channels is undertaken here by using HVM turbulence simulation. It is not always

\( \text{The method described in the work of Yang et al., }^{27,28} \text{ briefly revisited in the following, is based on the energy conservation equations, which—for the HVM model—read as} \)

\[
\frac{\partial \varepsilon_p^f}{\partial t} + \nabla \cdot \left( u_p \varepsilon_p^f + u_p \cdot P_p \right) = \left( P_p \cdot \nabla \right) \cdot u_p + n_p u_p \cdot E, \quad (4) \\
\frac{\partial \varepsilon_p^{ch}}{\partial t} + \nabla \cdot \left( u_p \varepsilon_p^{ch} + q_p \right) = - \left( P_p \cdot \nabla \right) \cdot u_p, \quad (5) \\
\frac{\partial \varepsilon_m}{\partial t} + \nabla \cdot \left( E \times B \right) = - j \cdot E, \quad (6)
\]

where \( \varepsilon_p^f = n_p \mu_p^2 / 2, \quad \varepsilon_p^{ch} = 1 / 2 \int d^3 v f_p (v - u_p)^2 = 3 n_p T_p / 2 \) and \( \varepsilon_m = B^2 / 8 \) are the flow, internal, and magnetic energy densities, respectively. Note that the kinetic pressure \( \varepsilon_p^f = 1 / 2 \int d^3 v f_p v^2 = \varepsilon_p^{ch} + \varepsilon_m. \) Moreover, the kinetic temperature \( T_p \) is implicitly defined as usual through the velocity distribution function (VDF). 2nd-order moment, does not imply thermal equilibrium.

In Eqs. (4)–(6), the pressure tensor \( P_p \) and the heat flux \( q_p \) are defined as follows:

\[
P_p = \frac{1}{2} \int d^3 v (v - u_p) (v - u_p) f_p, \quad (7) \\
q_p = \frac{1}{2} \int d^3 v (v - u_p)^2 (v - u_p) f_p. \quad (8)
\]

To appreciate the conversions between different types of energies, Eqs. (4)–(6) are averaged on the spatial domain to get

\[
\frac{\partial \langle \varepsilon_p^f \rangle}{\partial t} = \langle j_p \cdot E \rangle + \langle (P_p \cdot \nabla) \cdot u_p \rangle, \quad (9) \\
\frac{\partial \langle \varepsilon_p^{ch} \rangle}{\partial t} = - \langle (P_p \cdot \nabla) \cdot u_p \rangle, \quad (10) \\
\frac{\partial \langle \varepsilon_m \rangle}{\partial t} = - \langle j \cdot E \rangle, \quad \langle j \rangle, \langle j \rangle (E) \rangle \text{ not zero but reduces to the electromagnetic field performed on electrons } \langle j \rangle, \langle j \rangle (E) \rangle \text{ does not reduce to a simple time derivative. Equations (9)–(11) suggest that the term } \langle j \rangle, \langle j \rangle (E) \rangle \text{ controls the transfer between fluid and magnetic energy, while } \langle (P_p \cdot \nabla) \cdot u_p \rangle \text{ transforms flow energy into internal energy.} \]

This last term can be further decomposed as

\[
(P_p \cdot \nabla) \cdot u_p \parallel = P_p \delta_{ij} + \Pi_p \parallel D_p, \quad (12)
\]

where \( P_p \parallel = P_p \delta_{ij} + \Pi_p \parallel D_p, \parallel j_p = \nabla \cdot u_p, \quad \Pi_p = \nabla \cdot u_p, \quad D_p = \delta_{ij} + \delta_{ij} / 2 - \delta_{ij} \delta_{ij} / 3. \) In the last expression, \( \delta_{ij} \) is
Kronecker’s delta and \( \partial_i \) denotes the partial derivatives with respect to the \( i \)-th spatial coordinate. The two terms at the right-hand side of Eq. (12) can be associated with plasma dilatations and compressions and with the trace-less pressure tensor (also including off diagonal pressure terms), respectively. Hereafter, \( P \beta \) and \( \Pi_{\|} : D_p \) are briefly denoted as the \( P \)-\( \beta \) and \( P \)-\( D \) terms, respectively. Both the terms appear with a negative sign in Eq. (10), thus implying that negative values of \( P \beta \) and \( \Pi_{\|} : D_p \) have the effect of increasing the proton internal energy. Moreover, both terms are not positive definite, and hence, their signs are decisive to discriminate the direction of the energy transfer between flow and internal energy.

The idea adopted in recent studies\(^ {27,28,33} \) is that the \( P \)-\( D \) term can provide significant insights into the mechanisms that transfer energy toward smaller scales, where energy is eventually dissipated. Indeed, in a collisional plasma (MHD framework), the traceless pressure-tensor terms are usually related to viscous dissipation.\(^ 4^1 \) Investigating whether this connection holds also in a weakly collisional system can be hence useful to understand the nature of dissipation although—it within the HVM model—collisions are not modeled and the system is formally reversible.\(^ {12–14} \)

### III. OVERVIEW OF THE NUMERICAL SIMULATIONS

Equations (1)–(3) are integrated in a 2.5D–3V phase space domain. We retain all velocity space directions, while physical-space vectors are three-dimensional but depend only on the two spatial coordinates \((x, y)\). This double-periodic spatial domain, whose size is \( L_x = L_y = L = 2\pi \times 20d_{\text{pp}} \) is discretized with \( N_x = N_y = 512 \) grid points. The velocity domain is discretized with \( N_v = N_x = N_y = 71 \) points in the range of \( v = [-5v_{\text{th},p}, 5v_{\text{th},p}] \) \((i = x, y, z)\). Boundary conditions impose \( v_{\text{th},p} > 5v_{\text{th},p} \) where the proton thermal speed \( v_{\text{th},p} = \sqrt{k_B T_p/e} \) is directly related to the Alfven speed through \( \beta_p = 2v_{\text{th},p}/c_A^2 = 2 \).

The proton VDF is initially Maxwellian, with uniform unit density. A uniform background out-of-plane magnetic field \( B_0 = B_0 e_z \) \((B_0 = 1)\) is initially imposed. The equilibrium is perturbed by imposing an initial 2D spectrum of Fourier modes, equi-partitioned between proton bulk velocity and magnetic field, and having an amplitude such that \( \delta B_{\text{rms}}/B_0 = \delta n_{\text{rms}}/n_A = 1/2 \). The injection is random in perpendicular phases and in the wave-number range of \( k \in [0.1, 0.3] \), where \( k = mk_0 \) with \( 2 \leq m \leq 6 \) and \( k_0 = 2\pi/L \). Neither density fluctuations nor parallel perturbations are introduced \((\delta n_p = \delta n_{\text{pp}} = \delta B_z = 0)\). A small resistivity \( \eta = 1.5 \times 10^{-5} \) is accurately introduced to avoid numerical instabilities.

Before focusing on the \( P \)-\( D \) analysis, we conclude this section by describing the simulation results concerning the onset of turbulence at proton inertial scales.

Owing to the presence of strong nonlinearities, the initial relatively large-scale perturbations couple and produce fluctuations at smaller scales. This cascade-like activity can easily be appreciated by examining the evolution of \( \langle \delta B_i^2 \rangle \) \([\text{Fig. 1}(a)]\), which represents a standard proxy for highlighting the presence of nonlinear couplings. The \( \langle \delta B_i^2 \rangle \) peak, occurring at \( t^* = 30Q_{\text{pp}}^{-1} \), is associated with the most intense turbulent activity in the numerical simulation.\(^ {34} \) Note that the time \( t^* \) corresponds to about 0.6\( t_{\text{pb}} \) where the nonlinear time is \( t_{\text{pb}} = L/\delta u(L) \), in which \( L \) is the turbulence correlation length and \( \delta u(L) \) is the fluctuation amplitude at \( L \). In our case, \( L \approx L/5 \) and \( \delta u(L) \approx \delta u_{\text{rms}} \). The generation of small-scale fluctuations can be recognized by looking at the omnidirectional (perpendicular) power spectral densities (PSDs), reported in Fig. 1(b), as a function of \( k_{\|} \). PSDs have been computed by averaging in the temporal range \( \Delta t_{\text{psd}} \in [20, 40] \) which corresponds to a nearly constant \( \langle \delta B^2 \rangle \) and is shown in panel (a) with a gray shaded area. Contour plots of \( j_{\|}(x, y) \) and \( \epsilon_{\|}(x, y) \) at the temporal instant \( t = t^* \).

The real-space picture of out-of-plane electric current \( j_{\|}(x, y) \) at \( t = t^* \) is reported in Fig. 1(c). This contour plot confirms a highly structured and inhomogeneous current density pattern, with strong current sheets and magnetic islands. In, or near, these strong current sheets, which themselves are embedded in a dynamic and strongly turbulent region, complex phenomena such as magnetic reconnection, energy dissipation, and other kinetic effects\(^ {27,43–45} \) are found. Intense current regions are often in close proximity to regions in which the plasma VDF is far from the equilibrium Maxwellian shape.\(^ {46–50} \) The presence of non-Maxwellian features in the proton VDF can also be associated with the entropy velocity-space cascade.\(^ {11–53} \) To quantify deviations of the proton VDF from the associated Maxwellian \( f_{\text{eq}} \), we here adopt the \( \epsilon_{\|} \) parameter\(^ {46} \)

\[
\epsilon_{\|}(x, t) = \frac{1}{n_p} \sqrt{\langle (f_{\|} - f_{\text{eq}})^2 \rangle d^3 v},
\]

1

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where \( f_M \) has the same density, bulk speed, and temperature of \( f_p \). The temporal evolution of \( \langle e_p \rangle(t) \), reported in Fig. 1(a), highlights that the generation of non-Maxwellian features grows together with the increase in the current activity (i.e., magnetic field gradients and small-scale fluctuations). The \( e_p \) parameter finally saturates at a nearly constant value. The contour plot of \( e_p(x, y) \) at \( t = t^* \), reported in Fig. 1(d), demonstrates a qualitative correlation of \( e_p(x, y) \) and \( j_z(x, y) \).

### IV. THE Pi-D ANALYSIS: ENERGY CONVERSION AND TEMPERATURE INCREASE

#### A. Bookkeeping the internal energy

In order to understand the energy transfer mechanisms occurring in a turbulent plasma, it is useful to apply the Pi-D method on the simulation described in Sec. II.

By integrating on time Eq. (10) and using Eq. (12), one gets

\[
\Delta \langle e_p^{\text{th}} \rangle(t) = -\int_0^t dt \langle \Pi_p : D_p \rangle(t) - \int_0^t dt \langle P_p \theta_p \rangle(t), \tag{14}
\]

where \( \Delta \) denotes the variation with respect to the initial values. Figure 2 reports the temporal evolution of the cumulative integrals of \( -\langle \Pi_p : D_p \rangle \) and \( -\langle P_p \theta_p \rangle \). These integrals have been numerically evaluated through the trapezoidal rule. The Pi-D term dominates over the pressure-dilatation term in the internal energy increase, suggesting that the cascade process remains quasi-incompressible. Since Eulerian simulations are almost noise-free, we can also report the evolution of the internal energy \( \Delta \langle e_p^{\text{th}} \rangle(t) = 3 \Delta (n_p T_p) / 2 \). According to Eq. (10), the internal energy variation should be exactly equal to the cumulative integral of \( -\langle P_p \theta_p \rangle \). The slight difference between these two quantities is mainly due to the level of accuracy for the energy conservation recovered in the numerical simulation (~0.5%).

It can also be shown that the Pi-D term has a direct effect on the temperature growth. This effect can be highlighted by writing

\[
\frac{3}{2} n_p \frac{\partial (T_p)}{\partial t} \simeq \langle \Pi_p : D_p \rangle, \tag{17}
\]

which fully clarifies the behavior observed in Fig. 2.

On the other hand, the evolution of the second term in Eq. (15), namely, \( 3 \Delta (n_p T_p) / 2 \), is similar to \( P \theta \) as it can be expected since eventual correlations between density and temperature can be understood in terms of compression and dilatation.

Also note that, in the current simulation, the average temperature increases without showing a saturation regime. The evolution of correlated density-temperature fluctuations instead saturates in the first stages of the numerical simulation. This is probably due to the finite level of compressivelike structures contained in the initial perturbations. This effect may possibly be understood in terms of nearly incompressible MHD theory. Detailed analysis of the effects of different initial conditions, namely, by (i) introducing a different level of initial compressibility and by (ii) changing the value of \( \beta_p \), which also regulates the level of compressive activity, will be the content of a future work.

#### B. Role of \( j \cdot E \)

In the initial stage of the simulation and up to the peak of the turbulence, \((t \approx t^*) \int dt (\langle -j_p \cdot E \rangle \) is positive, associated with the initial increase in magnetic fluctuation energy at the expense of flow energy (a feature familiar from MHD simulations). This evolution does not compare well with the temperature growth in Fig. 2, suggesting that, despite being adopted frequently as a proxy for heating and dissipation, the electromagnetic work is not globally correlated with the temperature increase. This information is already partially contained in Eqs. (9)–(11), even though—in principle—there could be transfers between magnetic and internal energies, through the intermediary of the kinetic energy.

For the sake of completeness, in Fig. 2, we also report the cumulative integral of \( -\langle j_p \cdot E \rangle \). Compared to the integral of total \( -\langle j \cdot E \rangle \), the difference represents the physical work that is done on electrons. It is worth highlighting that here we are focusing on global (averaged)
quantities, while \( j \cdot E \) is locally important to identify sites of kinetic activity and where heating occurs.48,56,57

C. Spatial distributions and correlations

Figure 3 shows the contour plots of \(-\Pi_p : D_p\) (a), \(Q_o = \alpha^2 / 4(\omega^2)\) (b), \(Q_j = j^2 / 4(f^2)\) (c), and \(Q_D = D_p : D_p / 4(D_p : D_p)\) (d), at the time instant \( t = t' \), \( Q_o \), \( Q_j \), and \( Q_D \) estimate the relative strength of rotation, current sheets, and strain, respectively.

Our finding supports the recent works27,28,33 where a coarse-grained correlation between intense \( Pi-D \) event regions as well as strong current structures and high vorticity and velocity strain regions has been described. These correlations can be also appreciated by looking at Fig. 4, where the conditional averages of \(-\Pi_p : D_p\) with respect to \( Q_o \) (a), \( Q_j \) (b), and \( Q_D \) (c) are reported: stronger \( Pi-D \) event regions are found near regions of strong fluid vorticity and strain, with a weaker correlation evident near regions of current enhancement. Although we are not modeling irreversible dissipation, the correlation between \( Pi-D \) and \( Q_D \)—usually related to viscosity in collisional plasmas—suggests a connection between two intrinsically different physical systems (collisionless and collisional).

To further support the idea that the plasma heating is an inhomogeneous process,54 Fig. 5 shows the probability density function (PDF) of \(-\Pi_p : D_p\). The \( Pi-D \) PDF is far different from the Gaussian expectation, confirming the presence of intermittency. The \( Pi-D \) PDF is also skewed or slightly elongated in the positive direction of \(-\Pi_p : D_p\), i.e., in the direction of net increase in the internal energy, as indicated by the positive sign of the \(-\Pi_p : D_p\) average.

Since Eulerian Vlasov simulations are almost noise-free, we can reveal whether the \( Pi-D \) term is correlated with kinetic characteristics of the proton VDF. This aspect can also be addressed with PIC simulations, but only when a quite large number of particles per cell are adopted.58–60 otherwise, several of the features shown below are masked due to the presence of numerical noise.50 Figure 6 shows the joint PDF of the total temperature \( T_p \) (a), temperature anisotropy \( 1 - T_{p\perp}/T_p \) (b), and \( Q_o \) (c).
related to dissipation has been observed in several simulation studies, and is an important facet of intermittency in plasma turbulence.

**D. Role of local transport terms**

To further support the fact that local effects are important in the energy transfer toward smaller scales, where dissipation is thought to occur, we conclude this section by selecting two vertical cuts in the spatial domain of the numerical simulation that crosses the strong current sheet.

Figure 7 shows these one-dimensional cuts, taken at $x = 41.7d_0$ (left column) and $y = 122.7d_0$ (right column). Each row reports, from top to bottom, (a) magnetic field components $B_x$, $B_y$, and $B_z$; (b) current density $j$; (c) proton temperature $T_p$; (d) temperature anisotropy $1 - T_{p,\perp}/T_{p,\parallel}$; (e) non-Maxwellian measure $\epsilon_p$; (f) $j \cdot E$; (g) the heat flux divergence $\nabla \cdot \mathbf{q}_p$; (h) $-\Pi_p : D_p$; and (i) $-P_p \beta_p$.

In both the events, at least one component of the magnetic field changes its sign and the current density has a significant growth, characterized by a double-peaked structure in the left column, as also recently observed in Earth’s magnetosheath. The proton temperature also increases close to the current density peaks. In the left-column panel, plasma is hotter inside the current sheet, while in the right-column panel, plasma heating is slightly displaced from the current sheet.

Both proxies of non-Maxwellianity reported in Fig. 7, namely, $1 - T_{p,\perp}/T_{p,\parallel}$ (d) and $\epsilon_p$ (e), present a broad structure where both parameters are significantly non-null. The presence of such large-scale structures of non-Maxwellian also causes the lack of correlation between the non-Maxwellianity parameters and $P-E$. Super imposed to these large-scale structures, quite strong peaks of $1 - T_{p,\perp}/T_{p,\parallel}$, collocated with the temperature increase and associated with a preferential perpendicular heating ($T_{p,\perp} > T_{p,\parallel}$), are also observed.

The $j \cdot E$ plot indicates that, although this term is not globally connected with the temperature increase, it can be locally significant, close to current sheets and regions where kinetic effects are important and intermittent dissipation may be active.

The heat-flux divergence panels (g) support the idea that this term can be locally important although it does not globally change the proton internal energy. Indeed, in the left-column panel, a strong peak close to current sheets is recovered, while in the right-column panel, $\nabla \cdot \mathbf{q}_p$ is almost flat.

Both $-\Pi_p : D_p$ (g) and $-P_p \beta_p$ (h) report variations across the structures, ranging from positive to negative values. Focusing on the $P-E$ term, it is apparent in the left-column panel that the positive peak of $P-E$ is displaced from the current structure and it is not associated with the temperature growth. On the other hand, in the right-

![FIG. 5. Probability Density Function (PDF) of $-\Pi_p : D_p$ (black) and Gaussian expectation (red dashed), at $t = t'$. The blue dashed vertical line is the average value of the PDF: $(-\Pi_p : D_p) = 7.5 \times 10^{-4}$](image)

(b, and non-Maxwellianity parameter $\epsilon_p$ (c) with $-\Pi_p : D_p$. Each joint PDF reveals an asymmetry in the positive direction $-\Pi_p : D_p$. A subtle correlation between regions of strong $P-E$ and areas where plasma is locally warmer and non-Maxwellian can be seen. However, the $P-E$ term is broadly distributed and therefore not strongly correlated with proton temperature heating nor with the plasma non-Maxwellianity, in part due to transport effects. Indeed, strong $\epsilon_p$, $T_p$, and $1 - T_{p,\perp}/T_{p,\parallel}$ are also associated with very weak $-\Pi_p : D_p$ regions.

The $P-E$ term hence reveals regions where kinetic processes may be at work, but the opposite is not true. This is probably caused by the fact that other terms can be locally important and may break the correlation. With regard to this last comment on correlations between $P-E$ and kinetic signatures, it should be borne in mind that, when examining the $P-E$ term at the local level or using in situ measurements, transport terms may also play a role. Some of these terms, which are related to divergence, move energy from one location to another without globally changing the form of energy.

Furthermore, both the magnetic field and the spatial derivatives of the bulk speed enter into the dynamics of the pressure tensor. Consequently, various kinetic and fluid signatures of dissipation, such as $P-E$, current, $j \cdot E$, vorticity, temperature anisotropy, and non-Maxwellianity, may be coarsely or regionally correlated but pointwise offset from one another. This type of regional clustering of quantities related to dissipation has been observed in several simulation
column panel, two weak negative $-\Pi_p : D_p$ peaks are found in correspondence with the current structure, while $P_p \beta_p$ slightly increases. Figure 7 helps to realize the complexity of truly understanding how energy is dissipated in a turbulent, weakly collisional plasma. It is largely established that temperature growth occurs in or around current sheets. The same holds for temperature anisotropy and $\epsilon_p$, although structures are wider. The transfer mechanism based on the $P_i-D_p$ term is somehow correlated with hotter plasma regions although other terms ($P_p \beta_p$ heat-flux) may locally play a role. In our opinion, this should motivate extremely accurate in situ measurements to better appreciate the role of these variables.

V. CONCLUSION

We have examined the production of internal energy in a collisionless plasma employing an Eulerian HVM scheme and the pressure strain formalism.

To our knowledge, a study of this type has not been reported using an Eulerian approach. Relative to previous reports based on PIC...
models, the HVM approach has the advantage that the proton velocity distribution is well resolved in three dimensions and for several thermal speeds. Therefore, we may have some confidence that the subtle interactions between the pressure tensor and the spatial derivative of fluid velocity entailed in the pressure-strain interaction are well-represented in the computed solution. The HVM method is much less "noisy" than PIC, especially with regard to particle counting noise associated with a finite particle number (see, e.g., discussion in the studies by Pezzi et al. and Haggerty et al.). This permits HVM to compute accurate structures in velocity space as required for the present study.

Enabled in this way, we have examined characteristic kinetic properties during turbulence evolution, near the time of maximum mean square current density, when nonlinear effects are the strongest. Indeed, we have observed at this time the emergence of strong coherent current structures in real space, representing intermittency and strongly non-Maxwellian features in the proton VDFs, as has been previously reported.37,38

One of the very basic demonstrations of the efficacy of the HVM simulation strategy, and of the physical relevance of the pressure-strain relations, is seen in Fig. 2. Here, we have shown that the time integration of the sum of the two volume-integrated pressure-strain effects—the pressure-dilatation $P\partial$ and the $P\partial D$ terms—provides a very accurate account of the change in internal energy due to the turbulence cascade. On the other hand, the electromagnetic work on particles, $j \cdot E$, does not correlate well with the global internal energy increase. This would be anticipated based on the analysis leading to Eqs. (9)–(11) as one observes that the charge flux of species interacts with the electric field only to change the species flow energy at the expense of magnetic energy.

The present study has also reported that the values of the pressure-strain energy conversion ($-\Pi_p : D_p$) are broadly distributed and skewed toward positive values associated with production of internal energy. Spatial maps furthermore indicate that strong $-\Pi_p : D_p$ values appear in sheet (or core)like patterns, indicating intermittent conversion of flow energy into internal energy. Both these distributions are typical of quantities related to the cascade and pathways to dissipation, such as the "local energy transfer" (LET)33,62,63 or $j \cdot E$ in various reference frames.37,39

It remains to draw some conclusion regarding the idea of "dissipation." A reasonable definition of dissipation may require the conversion of energy into internal energy. Leaving momentarily aside the role of collisions in energy conversion and introduction of irreversibility, the spatial concentration of $P\partial D$ observed here (and earlier in PIC simulations46) appears to be precisely what is required for a plasma physics adaptation of the Kolmogorov refined similarity hypothesis. In such a development, the concentration of dissipation is directly related to scaling of velocity increments, including multifractal behavior, etc. In this way, it appears likely that the pressure-strain interaction will enter prominently into more formal and more complete theories of plasma turbulence which remain to be developed.

The role of collisions in attenuating velocity-space structures and in securing irreversibility remains to be further explored.12–14 This need is particularly clear pertaining to the relationship of weak collisions to the strength of pressure-strain interactions since formal (viscous) closure relating the two is not available in the weak collisional case.

Furthermore, within a collisionless system, the internal energy growth is not uniquely related to the increase in the random particle energy since the internal energy may also increase in the presence of non-Maxwellian features such as accelerated particle beams. The VDF free-energy contained in such structures10 can be backward released in the form of ordered energy, such as microinstabilities without increasing the plasma entropy.7 Collisions would intrinsically inhibit the possibility to transfer back this from particles to field this energy since—dissipating these structures—they incessantly push plasma toward thermal equilibrium.7,15 This further level of complexity should motivate subsequent studies to provide insights into the strength of the physical ingredient that ultimately dissipates energy in an irreversible way.16

**ACKNOWLEDGMENTS**

This research was supported in part by NASA through MMS Theory and Modeling Program No. NNX14AC39G and the Parker Solar Probe project under Princeton Subcontract No. SUB0000165, and HSR Grant No. 80SC18K1648. This paper has received funding from the European Union’s Horizon 2020 research and innovation program under Grant Agreement No. 776262 (AIDA, www.aida-space.eu). O.P. and S.S. are partly supported by the International Space Science Institute (ISSI) in the framework of the International Team 405 entitled “Current Sheets, Turbulence, Structures and Particle Acceleration in the Heliosphere.” Numerical simulations discussed here have been performed on the Marconi cluster at CINECA (Italy), within the Project Nos. IsC53_RoC-SWT and IsC63_RoC-SWTB, and on the Newton cluster at the University of Calabria (Italy).

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