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Nonlinear regime of electrostatic waves propagation in presence of electron-electron collisions

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The effects are presented of including electron-electron collisions in self-consistent Eulerian simulations of electrostatic wave propagation in nonlinear regime. The electron-electron collisions are approximately modeled through the full three-dimensional Dougherty collisional operator [J. P. Dougherty, Phys. Fluids 7, 1788 (1964)]; this allows the elimination of unphysical byproducts due to reduced dimensionality in velocity space. The effects of non-zero collisionality are discussed in the nonlinear regime of the symmetric bump-on-tail instability and in the propagation of the so-called kinetic electrostatic electron nonlinear (KEEN) waves [T. W. Johnston et al., Phys. Plasmas 16, 042105 (2009)]. For both cases, it is shown how collisions work to destroy the phase-space structures created by particle trapping effects and to damp the wave amplitude, as the system returns to the thermal equilibrium. In particular, for the case of the KEEN waves, once collisions have smoothed out the trapped particle population which sustains the KEEN fluctuations, additional oscillations at the Langmuir frequency are observed on the fundamental electric field spectral component, whose amplitude decays in time at the usual collisionless linear Landau damping rate.

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I. INTRODUCTION

In modeling the kinetic dynamics of a plasma, the collision-free approximation is usually considered. In many situations (like, for example, astrophysical or laboratory plasmas), particle collisions are, in fact, very rare, but the reason why collisionality is neglected most of the time is related more to the fact that collisions are hard to model, than to a justified physical approximation. As a matter of fact, from the kinetic point of view, the range of low collisionality can be significantly different from that of null collisionality. This is mainly due to the fact that the kinetic dynamics of a plasma is determined by the details of the particle distribution function in velocity space. For example, as recently shown in Ref. 1, slight departures from a Maxwellian can produce significant modifications in the dispersion relation of electrostatic waves, which involve both the real and the imaginary part of the wave frequency. Since particle collisions work to restore thermal equilibrium, it is clear that their effect can eventually change completely the features of the kinetic dynamics of a plasma, even in situations where collisionality can be considered “a priori” weak.

Of interest here are physical situations where collisionality is non-negligible (this allows removing the restriction to the collision-free or Vlasov approximation), but weak enough that the system, or a part of it, is far from the fluid plasma regime, where the fluid plasma can be everywhere considered to be fairly close to thermodynamic equilibrium. In these conditions, kinetic dynamics and collisionality are in competition between each other: while the first works to produce deformations of the particle distribution function, the latter tends to restore the Maxwellian configuration. The evolution of the plasma is, therefore, the result of the complex combination of these two effects.

The natural operator to model Coulombian interactions in a plasma is the Landau operator, a nonlinear integro-differential operator of the Fokker-Planck type and three-dimensional in velocity space.2–4 The Landau integral conserves mass, momentum and energy and obeys a H theorem for the entropy growth. Modeling particle collisions through the full three-dimensional Landau integral is, unfortunately, a hard goal to achieve, both from the analytical and the numerical point of view. As a consequence, simplified collisional operators are usually employed,5,6 which are easy to handle as compared to the Landau operator, but, of course, can capture only a part of the physics of collisions described by the complete Landau integral.

In this work, we model particle collisions through the Dougherty (DG) collisional operator.7 The DG operator is an “ad-hoc” nonlinear Fokker-Planck differential operator which describes diffusion in three-dimensional velocity space. Dougherty derived it in a phenomenological way, requiring good conservation properties (mass, momentum, and energy) and the existence of a H theorem for entropy.7–9 The DG operator is significantly less time demanding than the full Landau collisional integral. In fact, the computational time $t_c$ for 1D-3V (1D in physical space and 3D in velocity space) Eulerian simulations which include the full Landau operator scales as $t_c \sim N^7$ (where $N$ is the number of gridpoints, assumed, for simplicity, to be the same for each phase-space coordinate); for the DG operator, the scaling is $t_c \sim N^4$; this significant reduction of $t_c$ allows to run numerical experiments of the self-consistent electrostatic dynamics of a collisional plasma in 1D-3V geometry. In previous works,10–13 numerical simulations of electrostatic waves in collisional plasmas have been performed in a phase space of reduced dimensionality (1D-1V).
As the complete Landau description of particle collisions is not affordable, one should ask which minimal ingredients a simplified collision operator must have and which requirements it must satisfy to model a real plasma. In this perspective, in 2014 Pezzi et al.\textsuperscript{7} performed a detailed comparison between the Landau and the Dougherty collision operators, by means of Eulerian simulations, in the case of relaxation toward equilibrium of a spatially homogeneous field-free plasma in three-dimensional velocity space. From this analysis, it was shown that the return of the particle distribution function towards the Maxwellian shape and the collisional evolution of its relevant moments (temperature and entropy) are similar in the two cases (Landau and Dougherty), once an “ad hoc” time rescaling procedure has been performed. This time rescaling results, in practice, in dividing the plasma parameter $g$ in the DG operator by a factor $x \simeq 3.55$, whose value has been determined empirically from the numerical simulations. We point out that, due to the computational cost of the numerical approximation of the Landau integral, this analysis could not be performed in situations of self-consistent plasma evolution, not even in electrostatic approximation. Based on these considerations, in the present paper, we employ the DG operator to model particle collisions, rescaling the plasma parameter as discussed above and making the assumption that this procedure works to mimic the Landau integral also in self-consistent electrostatic situations. The goal of the present work is to present results from two different problems: the Landau and the Dougherty collisional evolution of its relevant moments (temperature and entropy) are similar in the two cases (Landau and Dougherty), and neglect electron-proton and proton-proton collisions, as their characteristic time is significantly longer than that for electron-electron interactions.\textsuperscript{3,12,13}

We consider the following dimensionless Dougherty-Poisson (DP) equations, in 1D–3V phase space configuration:

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + \frac{\partial \phi}{\partial x} \frac{\partial f}{\partial v_x} - \frac{\partial f}{\partial t}_{\text{coll}},$$

$$- \frac{\partial^2 \phi}{\partial x^2} = 1 - \int f \, dv;$$

where $f=f(x, v)$ is the electron distribution function, $\phi = \phi(x) = -dE/dx$ is the electrostatic potential ($E$ is the electric field), and $\partial f / \partial t_{\text{coll}}$ is the Dougherty collisional operator. Due to their inertia, the protons are considered as a motionless neutralizing background of constant density $n_0 = 1$. In previous equations, time is scaled to the inverse electron plasma frequency $\omega_{pe}$, velocities to the initial electron thermal speed $v_{th,e}$; consequently, lengths are normalized by the electron Debye length $\lambda_{De} = v_{th,e}/\omega_{pe}$ and the electric field by $\omega_{pe} m v_{th,e} / e$ ($m$ and $e$ being the electron mass and charge, respectively). For the sake of simplicity, from now on, all quantities will be scaled using the characteristic parameters listed above.

The Dougherty collisional operator\textsuperscript{7,8} has the following form:

$$\frac{\partial f}{\partial t}_{\text{coll}} = \nu(n, T) \frac{\partial}{\partial v_j} \left[ T \frac{\partial f}{\partial v_j} + (v - V) f \right];$$

here, $\nu(n, T)$ is the collision frequency,

$$\nu(n, T) = n_0 \frac{n}{T^{3/2}}; \quad n_0 = \frac{g \ln \Lambda}{2 \pi};$$

where $g = 1/n_{\Omega_{De}}^2$ is the plasma parameter, $\ln \Lambda \simeq -\ln x/3$ is the Coulombian logarithm, $x = 3.55$ is the scaling factor discussed previously, and the subscript $j$ indicates the $j$-th vector component. Moreover $n = \int d^3 v \, f$, $V_j = 1/n \int d^3 v \, v_j f$, $T = 1/3n \int d^3 v \, (v - V)^2 f$ are, respectively, plasma density, mean velocity, and temperature. These last quantities obviously depend on coordinate $x$, since $f=f(x, v)$, Einstein notation has been used in Eq. (3).

We solve numerically Eqs. (1) and (2) through a Eulerian code based on a finite difference scheme for the approximation of spatial and velocity derivatives of $f$ over the grid-points.\textsuperscript{17–19} Time evolution of the distribution function is approximated by using the splitting scheme proposed by Filbet et al.\textsuperscript{20} (see also Refs. 12 and 13 for details about the numerical algorithm). We employ periodic boundary conditions in physical space and $f$ is set equal to zero for $|v_j| > v_{\text{max}}$, where $v_{\text{max}} = 6v_{th,e}$. Phase space is discretized with $N_v = 128$ grid points in the physical domain $D_x = [0, L]$ and $N_v \times N_v \times N_v$ points in the three-dimensional velocity domain ($N_v = 101, N_v = N_v = 51$). Finally, the time step $\Delta t$ has been chosen in such a way to satisfy Courant-Friedrichs-Levy condition\textsuperscript{21} for the numerical stability of time explicit finite difference schemes.

**II. MATHEMATICAL AND NUMERICAL APPROACH**

We consider a plasma composed of kinetic electrons and motionless protons and analyze the dynamics of this system in electrostatic approximation. As discussed earlier, we model electron-electron collisions through the Dougherty operator$^{7,16}$ and neglect electron-proton and proton-proton collisions, as their characteristic time is significantly longer than that for electron-electron interactions.$^{3,12,13}$
III. NUMERICAL RESULTS

We present and discuss the results of kinetic Eulerian simulations in two different physical situations: the linear and nonlinear regime of the bump-on-tail instability and the excitation and propagation of the KEEN waves.

A. Bump-on-tail Instability

In this section, we focus on the process of bump-on-tail instability \(22,23 \) in a collisional plasma, in order to point out the role of collisions on the onset of the instability and on its nonlinear saturation. The initial electron distribution function considered for the numerical runs has the following form:

\[
 f(v_x, v_y, v_z, t = 0) = f_0(v_z) f_M(v_y) f_M(v_z),
 \]

where

\[
 f_0(v_z) = \frac{n_1}{(2\pi T_1)^{1/2}} \exp \left( -\frac{v_z^2}{2T_1} \right) + \frac{n_2}{(2\pi T_2)^{1/2}} \times \left[ \exp \left( -\frac{(v_z - V_0)^2}{2T_2} \right) + \exp \left( -\frac{(v_z + V_0)^2}{2T_2} \right) \right]
\]

\[
 f_M(v_y) = \frac{1}{(2\pi T)^{1/2}} \exp \left( -\frac{v_y^2}{2T} \right); \quad j = y, z
\]

with \( n_1 = 0.97, \quad n_2 = 0.015, \quad V_0 = 4.0, \quad T_1 = 1.0, \quad T_2 = 0.2 \). Moreover, \( f_M(v_y, v_z) \) is a normalized Maxwellian with temperature \( T = 1/n \int dv_y (v_y - V_y)^2 f_0(v_z) \). In these conditions, the plasma initially does not present any temperature anisotropy among the three velocity directions. Choosing an initial electron velocity distribution that is symmetric in \( v_z \) guarantees an initial state with no net plasma currents or magnetic fields. \(23\)

At \( t = 0 \), we perturb the system with a sinusoidal density perturbation of amplitude \( A_1 \sim 5.6 \times 10^{-4} \); we set the length of the spatial domain \( L \sim 22 \), in such a way to excite the most unstable wavenumber (the one with the largest growth rate) \( k^2 = 2\pi/L \sim 0.28 \), whose value has been predicted through a linear Vlasov solver, which computes numerically the roots of the electrostatic dielectric function. This density perturbation produces (through Poisson equation) an initial sinusoidal electric field of amplitude \( E_1 \sim 2 \times 10^{-3} \). Figure 1 shows \( f_0 \) as a function of \( v_z \); here, the vertical red-dashed line represents the value of the wave phase speed \( v_{\phi} \), which clearly falls in the unstable region where \( df_0/dv_z \mid_{v_z = v_{\phi}} > 0 \).

Figure 2(a) displays the time evolution of the logarithm of the fundamental electric field spectral component \( E_k \) (where \( k = k \)), normalized to its initial value (\( \log (E_k(t)/E_k(0)) \)), for a collisionless simulation. In the early stage of the system evolution, a linear exponential growth of the wave amplitude is observed with growth rate \( \gamma_{\text{obs}} = 7.29 \times 10^{-2} \); this value is in good agreement with the theoretical expectation obtained through a numerical linear Vlasov solver \( \gamma_{\text{th}} = 7.46 \times 10^{-2} \) (red-dashed line). Later in time, nonlinear effects come into play and arrest the exponential growth; in this regime, the wave amplitude displays nearly periodic oscillations around an almost constant saturation level. These oscillations are driven by particle trapping processes \(24,25 \) and typical vortical structures are generated in the longitudinal \( (x - v_x) \) phase space, in the velocity range around \( v_{\phi} \).

When collisions are taken into account, the system evolution can change significantly. In Figure 2(b), we show a collisional simulation with \( \nu_0 \sim 2.17 \times 10^{-3} \). For such value of the collision frequency, the linear growth of the wave amplitude remains close to exponential with a growth rate somewhat less than that for the collisionless case. This suggests that, in this scenario, the damping rate due to collisions is lower than the growth rate of the instability, thus showing that collisions are too weak to prevent the instability onset. However, the nonlinear saturation of the instability is evidently affected by collisions. In fact, from Figure 2(b), one notices that the saturation amplitude is decreased with respect to the collisionless case and that the electric oscillations are significantly damped after the saturation of the instability, as collisions work to smooth out the trapping structure and to drive the particle distribution towards the equilibrium Maxwellian shape. Additional runs with larger values of \( \nu_0 \) (not presented here) show how also the linear phase of the system evolution is modified in the strong collisional regime and eventually the onset of the instability is completely prevented.

Finally, we have evaluated from the simulations the entropy \( S = -\int f \ln f dv \). In Fig. 2(c), we compare the entropy growth, defined as \( \Delta S = [S(t) - S(t = 0)]/S(t = 0) \), for the collisionless case (black solid line) and for the weakly collisional case with \( \nu_0 = 2.17 \times 10^{-3} \) (red solid line). Since the collisionless Vlasov system is an iso-entropic system, the small entropy growth (\( \approx 0.15\% \)) recovered in the collisionless simulation is obviously due to numerical effects (filamentation). On the other hand, in the collisional case, the increase in entropy (about 10 times larger than the unphysical entropy growth for the collisionless simulation) is mainly due to the effect of collisions which drive the system towards thermal equilibrium, according to H theorem.

To conclude this section, in Fig. 3 we show the \( x - v_x \) contour plots (zoomed in the velocity range \( \approx v_{\phi} \)) of the distribution function evaluated at \( v_z = v_z = 0 \); the top/bottom row in this figure corresponds to the collisionless/collisional
case. We plot the distribution function at two instants of time in the simulations ($\tau_1 = 80$ and $\tau_2 = 320$), indicated by the vertical solid-blue lines in Figs. 2(a) and 2(b); $\tau_1$ corresponds to the end of the exponential growth phase of the wave amplitude, while $\tau_2$ is picked in the nonlinear regime of wave propagation. In the top row of Fig. 3 (collisionless case), one recognizes (left panel) the vortical phase-space structure at $v_x \simeq v_y \simeq 3.5$, typical signature of particle trapping, which is persistent in time (right panel). A similar phase-space vortex (not shown here) is recovered at $v_x \simeq 3.5$; the two counter-propagating phase space trapping populations are associated with the standing plasma wave launched by the initial density perturbation and amplified by the bump-on-tail instability. In the bottom row of the same figure (collisional case), at $t = \tau_1$ [Fig. 3(c)], the vortex has a smaller velocity width as compared to the collisionless simulation; moreover, collisions prevent the generation of fine velocity scales and, at $t = \tau_2$ [Fig. 3(d)], the trapping structure has been almost completely smoothed out.

Figure 4 shows, in a semi-logarithmic plot, the dependence of the distribution function on $v_x$ (evaluated at a fixed spatial position $x_0$, and at $v_y = v_z = 0$) at the time instant $t = \tau_1$.
waves, we drive the plasma through an external electric field (black-solid line) is due to the fact that at time instant \( t = 1200 \), for \( \nu_0 = 0.0 \) (black-solid line) and \( \nu_0 = 2.17 \times 10^{-3} \) (red-solid line); the black-dashed curve indicates the corresponding Maxwellian.

\( t = 1200 \), for the collisional simulation (red-solid line) and the collisionless one (black-solid line). The point \( x_0 \) corresponds to the spatial position where the phase space vortex moving with positive velocity has its maximum velocity width. In the collisional case, thermal equilibrium has been almost restored by collisions, while, in absence of collisions, the distribution function still displays many strong deviations from the Maxwellian profile (represented by the black-dashed curve). We point out that the asymmetry of the velocity profile for the collisionless simulation in Fig. 4 (black-solid line) is due to the fact that at \( t = 1200 \), the two counter-propagating phase space trapping vortices are not exactly aligned in phase space (i.e., their centers are not in the same spatial location).

**B. KEEN waves**

For the simulations of KEEN wave excitation,\(^{14,15}\) we refer to a previous work by Cheng et al.\(^{26}\) According to these authors, the box length for this simulation is set \( L = 24.166 \). At \( t = 0 \) the plasma is spatially homogeneous with density \( n_0 = 1 \) and isotropic Maxwellian in velocities with temperature \( T = 1 \). In order to produce the excitation of KEEN waves, we drive the plasma through an external electric field of the form\(^{14}\)

\[
E_D(x,t) = E_0 g(t) \sin[k_0(x - v_\phi t)],
\]

where \( E_0 \) is the maximum driver amplitude, \( k_0 = 2\pi/L = 0.26 \) is the fundamental wavenumber, \( v_\phi = 1.42 \) and

\[
g(t) = \begin{cases} 
\sin(\pi/100) & t < 50 \\
1 & 50 \leq t < 150 \\
\cos[\pi(t - 150)/100] & 150 \leq t < 200 \\
0 & t \geq 200.
\end{cases}
\]

The external field is turned off after a time at which past experience indicates that optimal trapping of particles is achieved (i.e., an appropriate ratio of an electron trapping period for the external drive). We performed different simulations by varying the value of the plasma parameter \( g \) and consequently of \( \nu_0 \) (\( \nu_0 = 0.00, 3.23 \times 10^{-4}, 2.17 \times 10^{-3} \)), keeping fixed \( E_0 = 0.05 \).

Figure 5 shows the evolution of the first four electric field spectral components (with wavenumbers \( k_1 = k_0, k_2 = 2k_0, k_3 = 3k_0 \) and \( k_4 = 4k_0 \)), for \( \nu_0 = 0.00 \) (a), \( \nu_0 = 3.23 \times 10^{-4} \) (b), and \( \nu_0 = 2.17 \times 10^{-3} \) (c), respectively.

In the collisionless case [Fig. 5(a)], we recover one of the typical features of the KEEN waves\(^{14,15,26}\) While the driver is turned on, the energy injected into the fundamental wavenumber component (black line) flows also to the higher spectral components (red, blue and yellow solid lines). After the driver has been turned off, the resulting electric signal is composed by many wavenumbers, in a stable ratio one with another, thus departing significantly from the purely sinusoidal spatial shape of the driver field.

Figures 5(b) and 5(c) display the time evolution of the electric field spectral components in two different collisional plasmas, for \( \nu_0 = 3.23 \times 10^{-4} \) and \( \nu_0 = 2.17 \times 10^{-3} \), respectively.

Beginning with the behavior while under the drive, on comparing the behavior to that without collisions, the behavior seems quite straightforward. For the weakly collisional case of Fig. 5(b), in the initial phase of the system evolution (i.e., up to \( t = 200 \)), when the external driver is on, the excitation of the spectral components does not seem to be significantly affected by collisions, i.e., the early parts of Figs. 5(a) and 5(b) look much alike. On the other hand the response of Fig. 5(c) with strong collisions is much weaker.

![FIG. 5. Time evolution of the first four electric field spectral components for the simulations with \( \nu_0 = 0.0 \) (a), \( \nu_0 = 3.23 \times 10^{-4} \) (b), and \( \nu_0 = 2.17 \times 10^{-3} \) (c). The black-dashed curves in panels (b) and (c) indicate the theoretical Landau prediction \( \gamma_L \approx 3.40 \times 10^{-4} \) for Langmuir wave damping rate.](image_url)
Turning now to the behavior after the drive has stopped, a significant difference between the damping is apparent between the cases where the damping is zero (Fig. 5(a)), moderate (Fig. 5(b)), and strong (Fig. 5(c)). At the extremes, the collisionless KEEN behavior of Fig. 5(a) with its strongly persistent harmonics is in striking contrast to the highly collisional case of Fig. 5(c) where the fundamental is the only component which survives in the long time limit. For intermediate collision frequency ($\nu_0 = 3.23 \times 10^{-4}$) case of Fig. 5(b), in the time interval $200 \leq t \leq 550$, the higher harmonic electric field components decrease somewhat faster than the fundamental (as one might expect) at roughly constant rates, but then there occurs a fairly sudden and remarkable transition (for $500 \leq t \leq 600$) to a much lower decay rate for the fundamental and an increased decay rate for the higher (2, 3, 4) harmonics. Thus at late times only the fundamental component survives.

These late-time fundamental decay rates recovered for the two collisional cases (Figs. 5(b) and 5(c)) seem almost independent of the collision frequency. Through a careful analysis of the time signals, we realized that the oscillations on the fundamental wavenumber, observed for $t > 600$ in Fig. 5(b) and for $t > 200$ in Fig. 5(c), occur at the Langmuir frequency, which is larger than the frequency of the KEEN waves excited by the driver. The dashed curves in Figs. 5(b) and 5(c) represent the prediction for collisionless Landau damping rate, which fits clearly well the numerical results, for both the intermediate and strong collisional cases. In order to understand the origin of these Langmuir fluctuations, we performed the Fourier analysis of the electric signal, in the time interval in which the driver is still on; this analysis revealed that the Langmuir frequency has been driven by the driver itself, which pumps energy at the KEEN

![FIG. 6. Spectral energy of the fundamental electric field component as a function of frequency, for the stronger collisional case with $\nu_0 = 2.17 \times 10^{-3}$, computed in the time intervals $0 \leq t \leq 180$ (black curve), and $400 \leq t \leq 1200$ (red curve). The vertical black-dashed curve indicates the value of the Langmuir frequency of the fundamental wavenumber.](image)

![FIG. 7. $x - v_x$ contour plots of the electron distribution function (zoomed in the velocity range $\simeq v_0$), evaluated at $v_y = v_z = 0$, for simulations with $\nu_0 = 0.0$, $3.23 \times 10^{-4}$, $2.17 \times 10^{-3}$ (top, middle and bottom row, respectively) and at different times $t = 200, 320, 400$ (left, middle, and right column, respectively).](image)
frequency, with an additional small amount of energy at the Langmuir frequency on the fundamental. The excitation of this additional Langmuir oscillation by the driver is due to the fact that the external electric field is turned on and off quite abruptly (with sharp time gradient in its amplitude). These abrupt kicks on the plasma excite Langmuir fluctuations, since they are proper modes of the system. Presumably, a smoother ramping up and down of the driver field (see Ref. 28) would have eliminated this additional Langmuir oscillation, but it would have required a significantly longer time for the driving process.

To substantiate the conclusions above, in Fig. 6, we report the spectral energy of the fundamental component as a function of frequency, for the stronger collisional case with $\nu_0 = 2.17 \times 10^{-3}$, computed in the time intervals $0 \leq t \leq 180$ (black curve), in which the driver is still on, and $400 \leq t \leq 1200$ (red curve), in which the driver is off. As it can be seen in this figure, when the driver is on, the main KEEN frequency peak is observed together with a low energy peak at the Langmuir frequency (vertical dotted-dashed black line in the figure); on the other hand, when the driver is off, the KEEN fluctuations are killed by collisions and only the Langmuir peak is visible. Finally, the fact that these Langmuir oscillations decay at the collisionless Landau damping rate suggests that collisions, which strongly affect the evolution of the KEEN fluctuations, are negligible at higher Langmuir phase speeds, where the particle velocity distribution remains close to a Maxwellian during the simulation.

To conclude this section, in Fig. 7 we show the contour plots of the electron distribution function (evaluated at $v_y = v_z = 0$) in the longitudinal $(x - v_x)$ phase space, for simulations with $\nu_0 = 0.0, 3.23 \times 10^{-4}, 2.17 \times 10^{-3}$ (top, middle, and bottom row, respectively) and at different times $t = 200$, $320$, $400$ (left, middle, and right column, respectively). These contour plots clearly show how the phase space trapping structure, which is persistent in the collisionless simulation and sustain the KEEN fluctuations, is smoothed out by collisions as fast as $\nu_0$ increases.

IV. SUMMARY AND CONCLUSIONS

In this paper, we presented the results of self-consistent Eulerian kinetic simulations of the propagation of nonlinear electrostatic waves, in weakly collisional plasmas. Electron-electron collisions have been modeled through the Dougherty collisional operator for electron-electron collisions have been modeled through the Dougherty collisional operator for electron-electron collisions, in full three-dimensional geometry in velocity space. This allows the distribution function to explore the entire velocity domain, under the effect of collisions, and prevent unphysical effects coming from collapsing the velocity diffusion process in a single velocity direction.

We described numerically the onset and nonlinear saturation of the bump-on-tail instability (in its symmetric form) and the excitation and propagation of the so-called Kinetic Electrostatic Electron Nonlinear waves, in situations of intermediate range of plasma collisionality. In this way, we get rid of the restrictive collision-free assumption, keeping, however, the system dynamics far from the strong collisional fluid regime, where the plasma always remains at thermodynamic equilibrium. In other words, the physical regime of interest here is the one where kinetic effects, which tend to drive the system far from the thermodynamic equilibrium, and collisions, which tend to restore the Maxwellian configuration, compete and combine themselves, shaping the particle distribution function in a complex way.

For the case of the symmetric bump-on-tail instability, we noticed that the onset of the instability (and the exponential growth of the wave amplitude) is almost unaffected, for the value of collision frequency chosen in our simulations. On the other hand, the nonlinear saturation phase, in which the fluctuations are maintained at almost constant amplitude thanks to the phase-space deformation of $f$, is dramatically modified by collisions, which in time work to smooth out any departure of $f$ from Maxwellian and damp the wave amplitude.

Concerning the simulations of the KEEN waves, we found that, in presence of collisions, the trapping phase space structure created by the driver field is smoothed out. As a consequence, the KEEN fluctuations are dissipated in time. In the case of intermediate collisionality, the fundamental spectral component and its harmonics (we have shown the first four) survive for a while after the driver is turned off. We noticed that in the long time limit the fundamental component displays a residual energy at the Langmuir frequency and its amplitude decays in time at a rate in good agreement with the collisionless damping rate predicted by Landau in Ref. 27. As explained previously, this Langmuir fluctuation has been triggered by the external field during the driving process. In the case of stronger collisionality, again fluctuations on the fundamental component appear at the Langmuir frequency in the long time limit, while the higher spectral components at the KEEN frequency are now very rapidly smoothed out by collisions, right after the driver has been turned off. The fact that the late-time decay rate of the fundamental is independent of the collision frequency, being in agreement with the collisionless Landau damping rate, suggests that the wave dissipation due to collisions is less efficient than the Landau damping process at high Langmuir phase speeds, where the particle velocity distribution remains close to a Maxwellian. On the other hand, the presence of (even weak) collisions is critical for the survival of the KEEN fluctuations, since the smoothing of the particle velocity distribution induced by collisions prevent the existence of the KEEN mode itself.